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FACULTÉ DES SCIENCES

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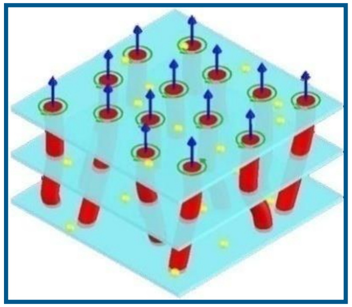
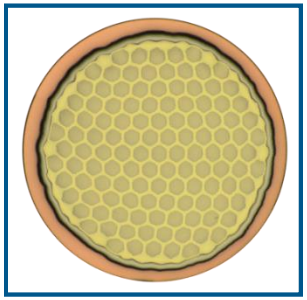
Superconductivity and its applications

Lecture 5





Carmine SENATORE

Group of Applied Superconductivity
Department of Quantum Matter Physics
University of Geneva, Switzerland

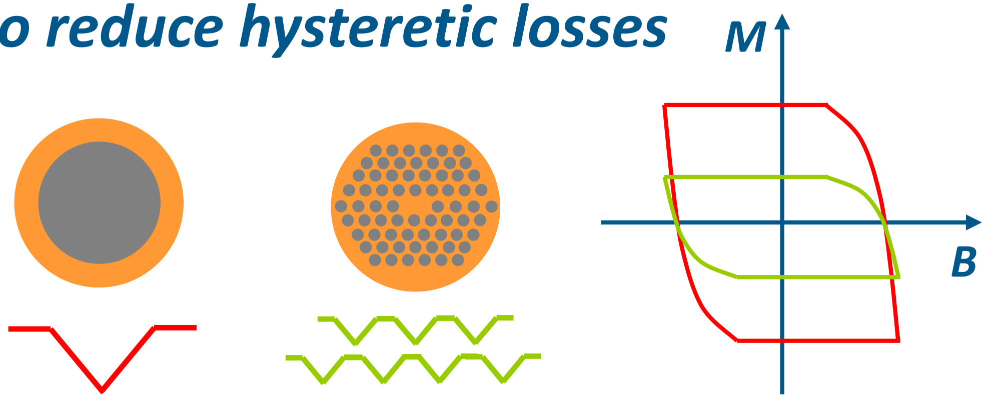
Previously, in lecture 4

1) In the presence of a current, vortices experience a force $F = J_{ext} \times \frac{B}{c}$ and vortex motion (flux flow) induces dissipation

$$\rho_{ff} = \rho_n \frac{B}{B_{c2}}$$

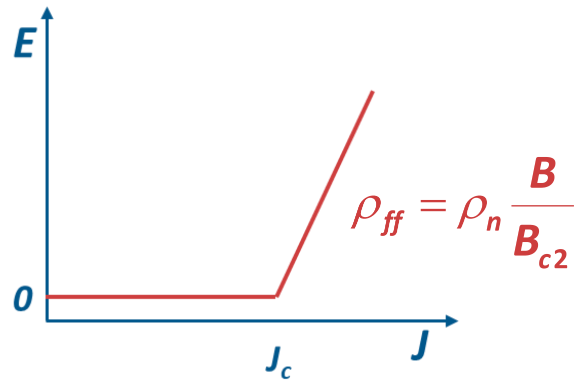
2) At T=0 and $J \leq J_c(B,T)$ an array of pinning centers may impede vortex motion (no dissipation)

3) In a wire the subdivision of the superconductor in fine filaments is required to reduce hysteretic losses

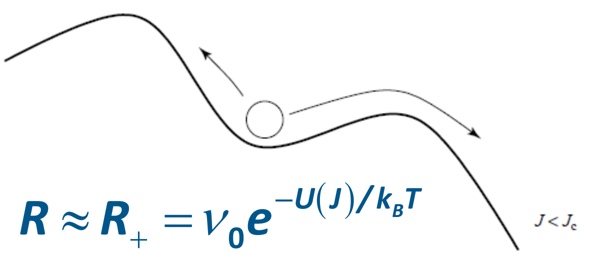


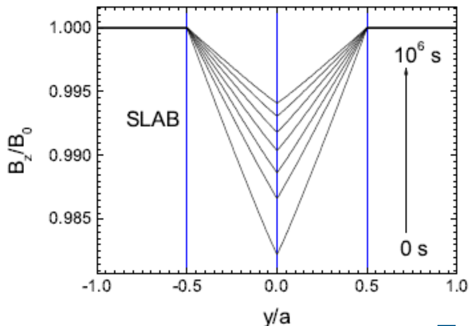
Previously, in lecture 4

4) E-J relation for a superconductor in the mixed state at T = 0: transition from critical state to flux flow



5) Flux motion is thermally activated at $T \neq 0$



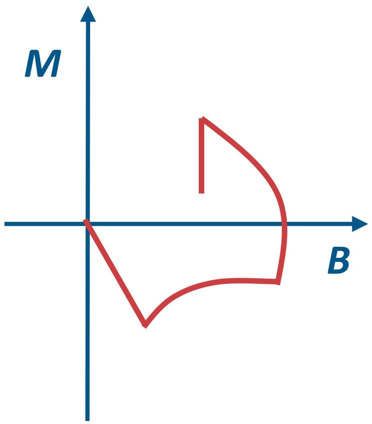


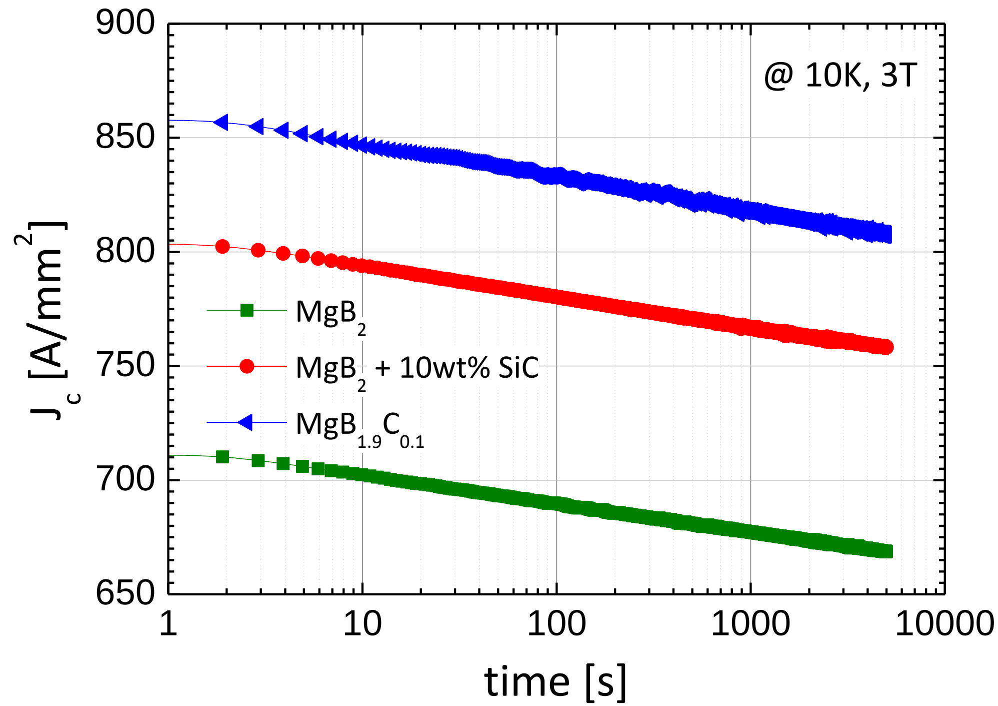
$M \propto J$

In the Anderson-Kim model $U = U_0\left(1 - \frac{J}{J_0}\right)$ and $J(t) = J_0\left[1 - \frac{k_B T}{U_0} \ln\left(\frac{t}{t_0}\right)\right]$

Determination of the pinning energy from magnetic relaxation experiments

$$M(t) = M_0\left[1 - \frac{k_B T}{U_0}\ln\left(\frac{t}{t_0}\right)\right]$$





The relaxation rate $S = -\frac{1}{M_0}\frac{dM}{d\ln t} = \frac{k_B T}{U_0}$ ***is inversely proportional to*** U_0

Measuring S as a function of B and T, we have an experimental access to $U_0 = U_0(B,T)$

Thermal activation and E-J curves

The Anderson-Kim model dependence $U = U_0\left(1 - \frac{J}{J_0}\right)$ *holds in the case of strong pinning (extended defects). And the E-J curve has the following expression*

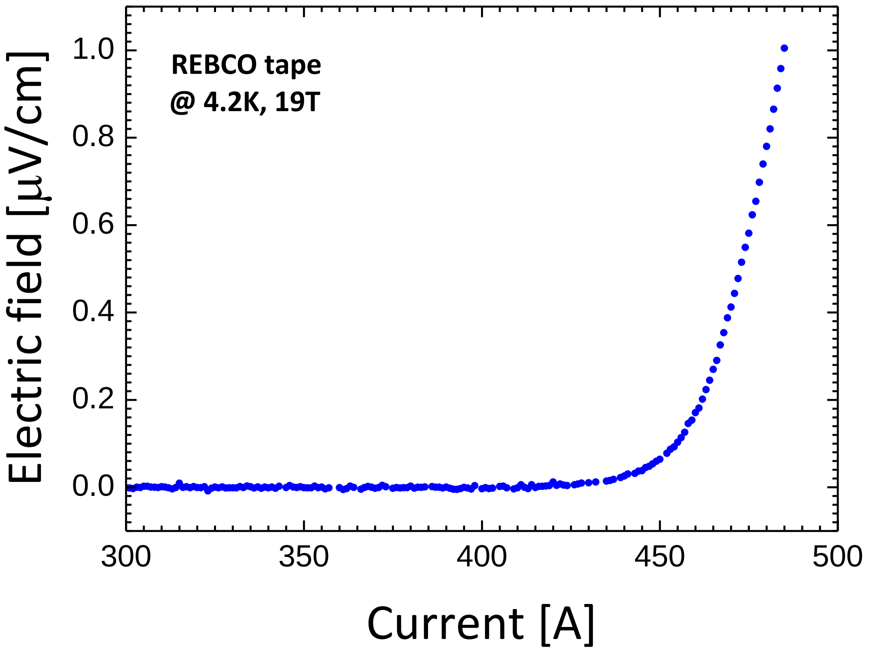
$$E = vB = Bv_0 \exp\left[-\frac{U(J)}{k_B T}\right] \quad \Rightarrow E \propto B\exp\left[-\frac{U_0}{k_B T}\left(1 - \frac{J}{J_0}\right)\right]$$

Other U(J) dependencies are theoretically predicted and experimentally observed when the elastic interactions among vortices cannot be neglected

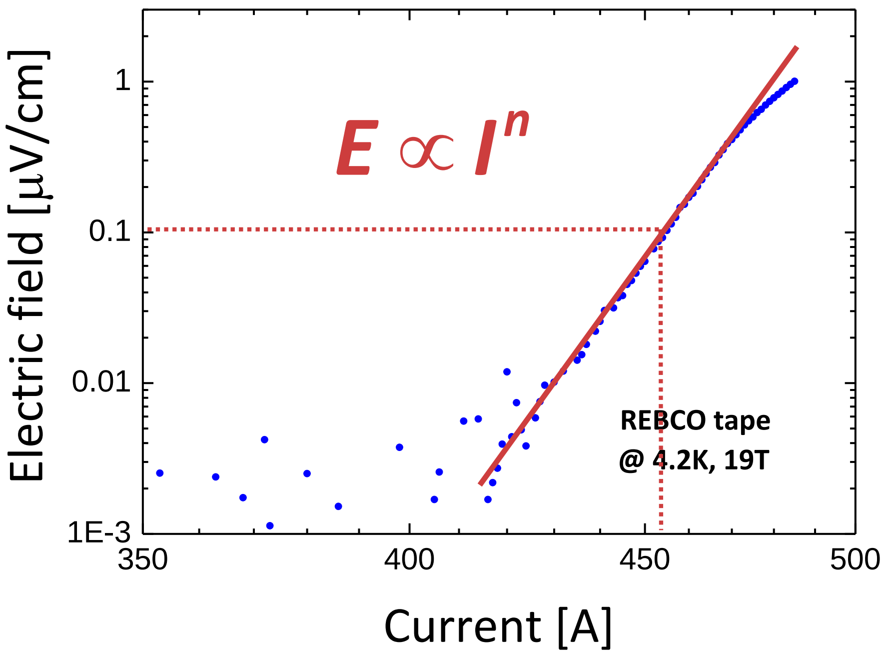
The one that is most often used is the "logarithmic barrier" $U = U_0 \ln\left(\frac{J_0}{J}\right)$

$$E \propto B\exp\left[-\frac{U_0}{k_B T}\ln\left(\frac{J_0}{J}\right)\right] \quad \Rightarrow \quad \boxed{E \propto \left(\frac{J}{J_0}\right)^{\frac{U_0}{k_B T}}}$$

Experimental V-I curves



Log(E)-Log(I) scale



The slope is $n = U_0/k_BT$

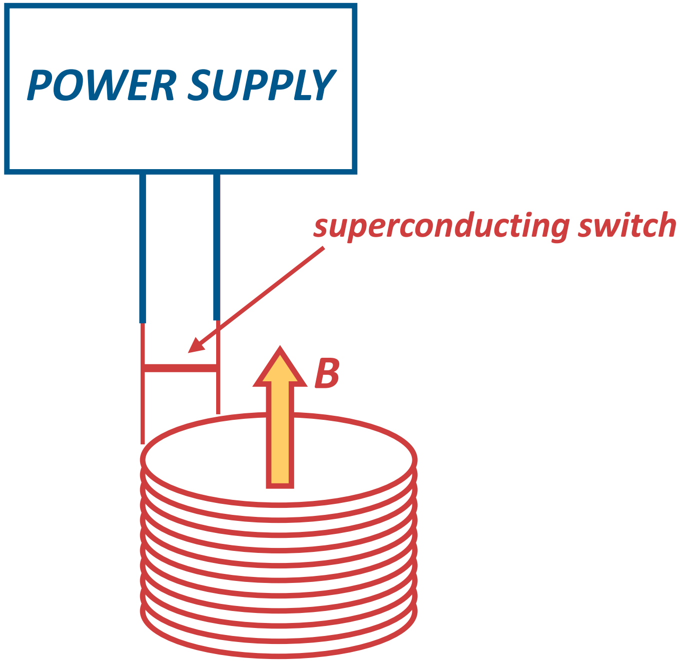
The two parameters that define a superconducting wire:

- *The critical current I_c is defined as the current corresponding to $E = 0.1\ \mu V/cm$*
- *The n-value is an indication of the pinning strength, but also of the homogeneity of the wire. Higher the n-value, better the pinning*

Persistent mode operation in a magnet

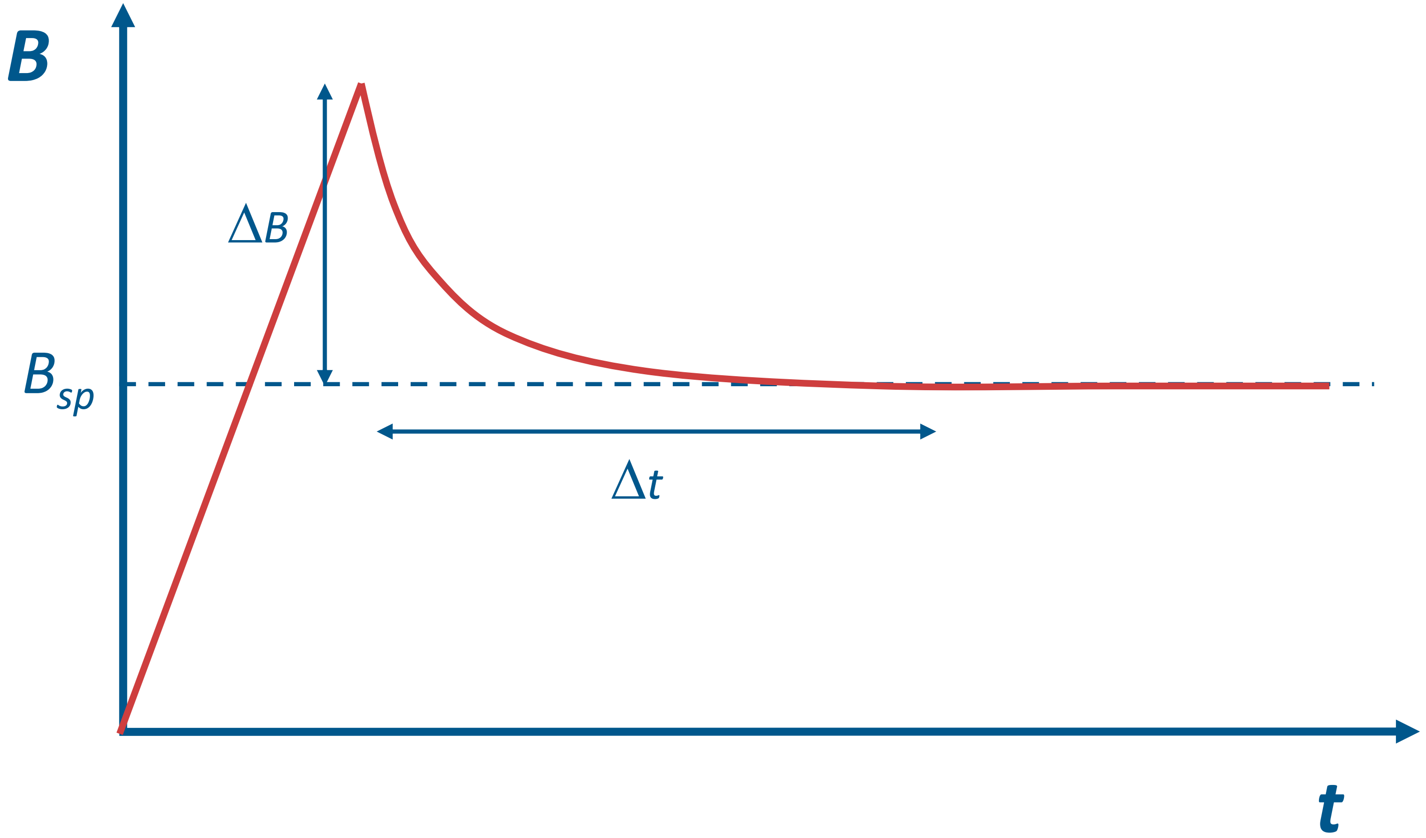
MRI and NMR magnets operate in persistent mode, without using a power supply

How does it work?



A high n-value (low magnetic relaxation) is necessary to operate in persistent mode

Persistent mode operation in a magnet



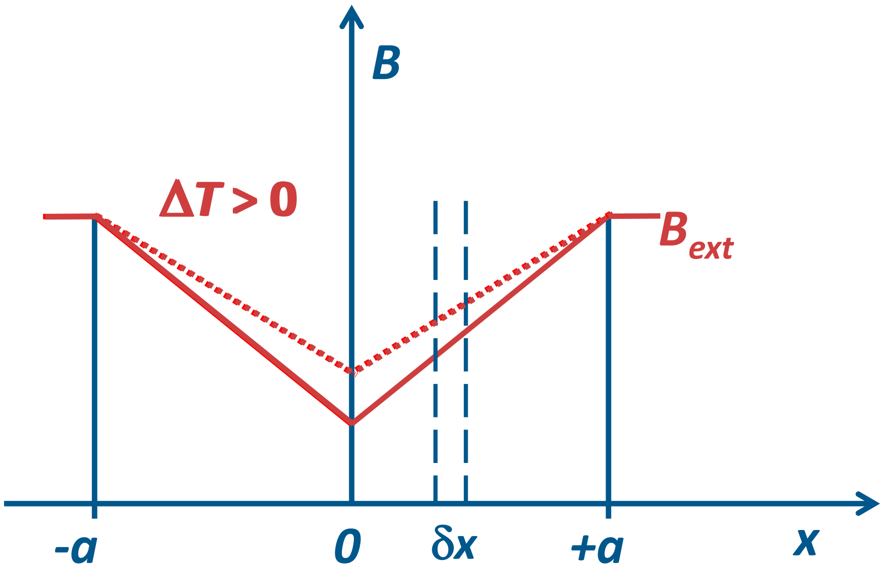
At the operation, the drift of the field is 10 ppb/hour

The field is reduced by half in ~6’000 years !!

Flux jumps and Thermal instabilities

In the adiabatic approximation

$$\begin{array}{ccc} \Delta T > 0 & \Rightarrow & \Delta J_c < 0 \\ \Uparrow & & \Downarrow \\ \Delta Q > 0 & \Leftarrow & \Delta E > 0 \end{array}$$



The heat generated in δx is

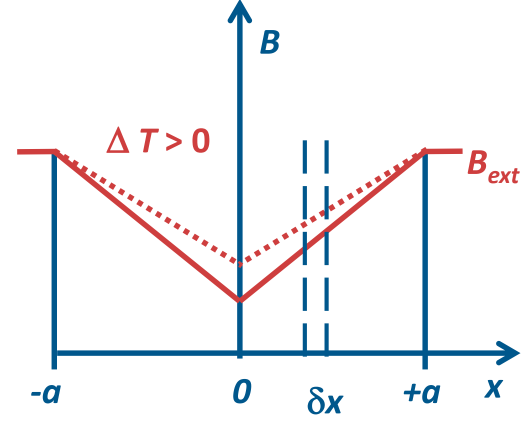
$$\delta q(x) = \int I(x) E(x) dt = J_c \delta x \delta\varphi(x)$$

The field profile for an infinite slab in parallel field is, for $0 < x < a$

$$B(x) = B_{ext} - \mu_0 J_c (a - x)$$

MKS

Flux jumps and Thermal instabilities



$$\delta q(x) = \int I(x)E(x)dt = J_c \delta x \delta\varphi(x)$$

$$B(x) = B_{ext} - \mu_0 J_c (a - x)$$

It follows

$$\delta\varphi(x) = \int_0^x \Delta B(x)dx = \int_0^x \mu_0 \Delta J_c (a-x)dx = \mu_0 \Delta J_c \left(ax - \frac{x^2}{2} \right)$$

The heat per unit of volume (averaged on the slab) is

$$\Delta Q = \frac{1}{a}\int_0^a \delta q(x)dx = \frac{1}{a}\int_0^a \mu_0 J_c \Delta J_c \left(ax - \frac{x^2}{2} \right)dx = \mu_0 J_c \Delta J_c \frac{a^2}{3}$$

Let's suppose a linear decrease of J_c with temperature

$$\Delta J_c = -J_c \frac{\Delta T}{(T_c - T_{op})}$$

MKS

Flux jumps and Thermal instabilities

Let's suppose a linear decrease of J_c with temperature

$$\Delta J_c = -J_c \frac{\Delta T}{\left(T_c - T_{op}\right)}$$

If ΔQ_{ext} is the initial perturbation, the heat balance for the slab is

$$\Delta Q_{ext} + \frac{\mu_0 J_c^2 a^2}{3\left(T_c - T_{op}\right)} \Delta T = c \Delta T$$

Because of the energy stored in the current, the effective specific heat is

$$c_{eff} = \frac{\Delta Q_{ext}}{\Delta T} = c - \frac{\mu_0 J_c^2 a^2}{3\left(T_c - T_{op}\right)}$$

c_{eff} can become zero $\Rightarrow$ ultimate thermal catastrophe !!

Flux jumps and Thermal instabilities

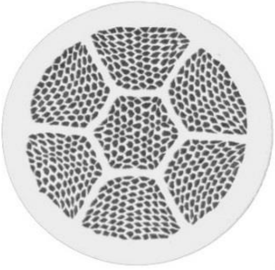
The stability condition is

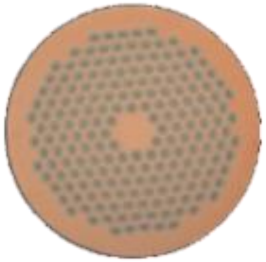
$$c_{eff} = c - \frac{\mu_0 J_c^2 a^2}{3\left(T_c - T_{op}\right)} > 0$$

A superconducting wire must be designed in such a way that

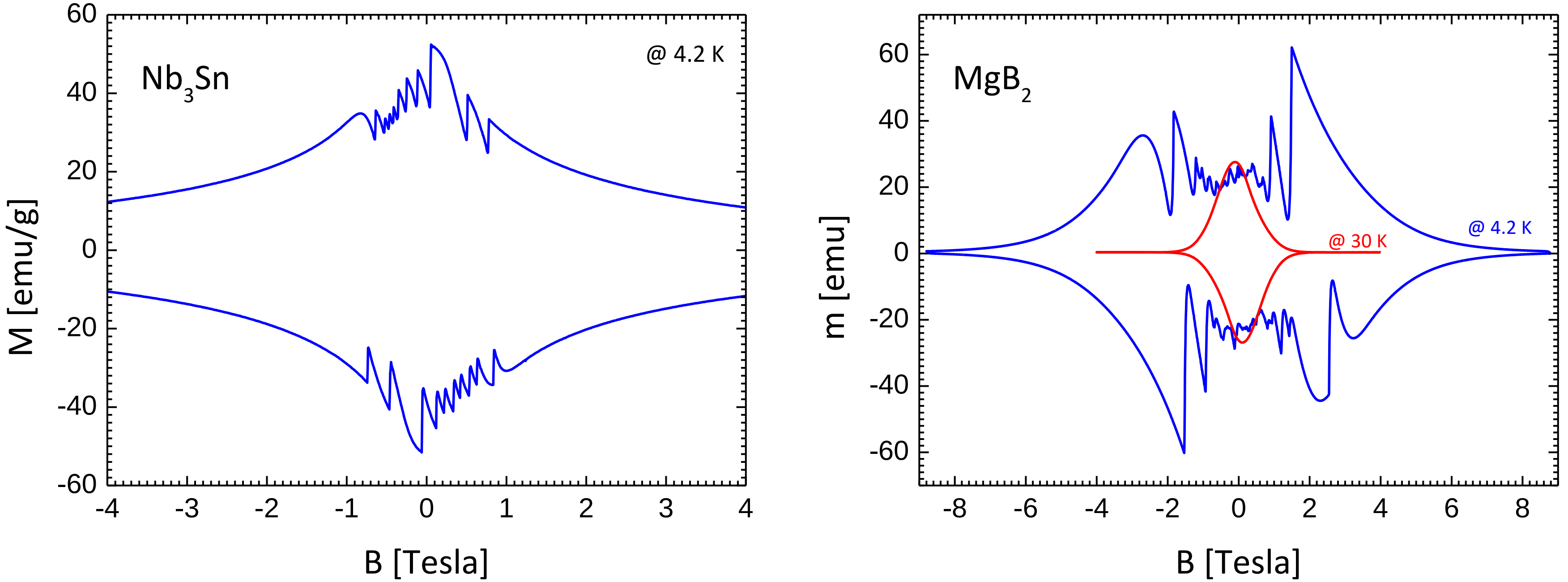
$$\frac{\mu_0 J_c^2 a^2}{c\left(T_c - T_{op}\right)} < 3$$

And this demands the subdivision of the superconductor in fine filaments



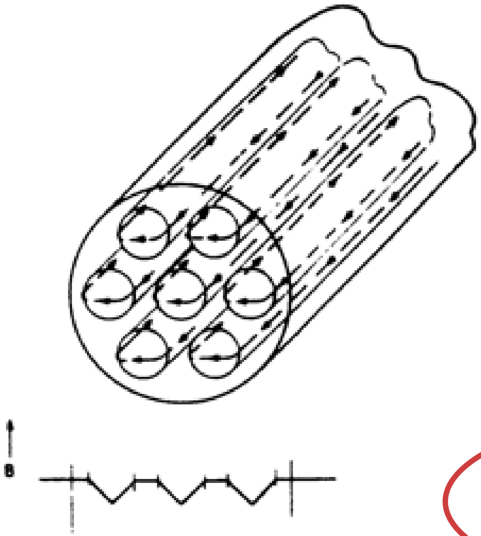


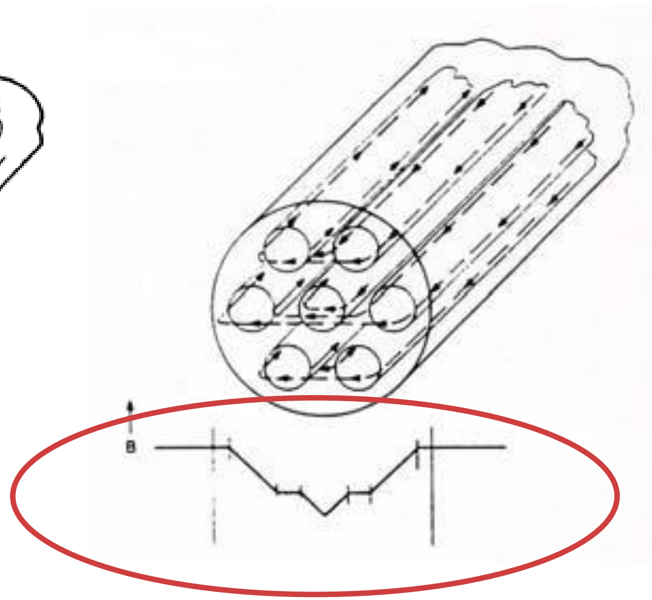
Flux jumps and Thermal instabilities

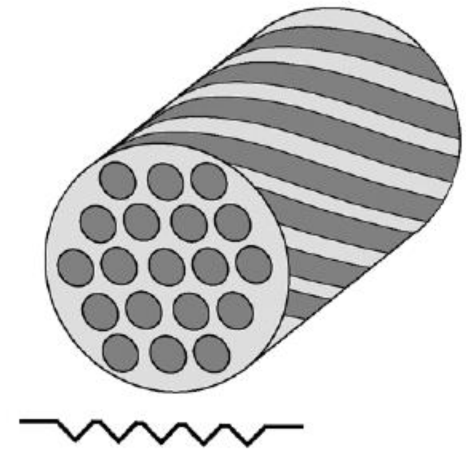


Stability criterion
$$\frac{\mu_0 J_c^2 a^2}{c\left(T_c - T_{op}\right)} < 3$$

Multifilamentary wires, coupling and twisting





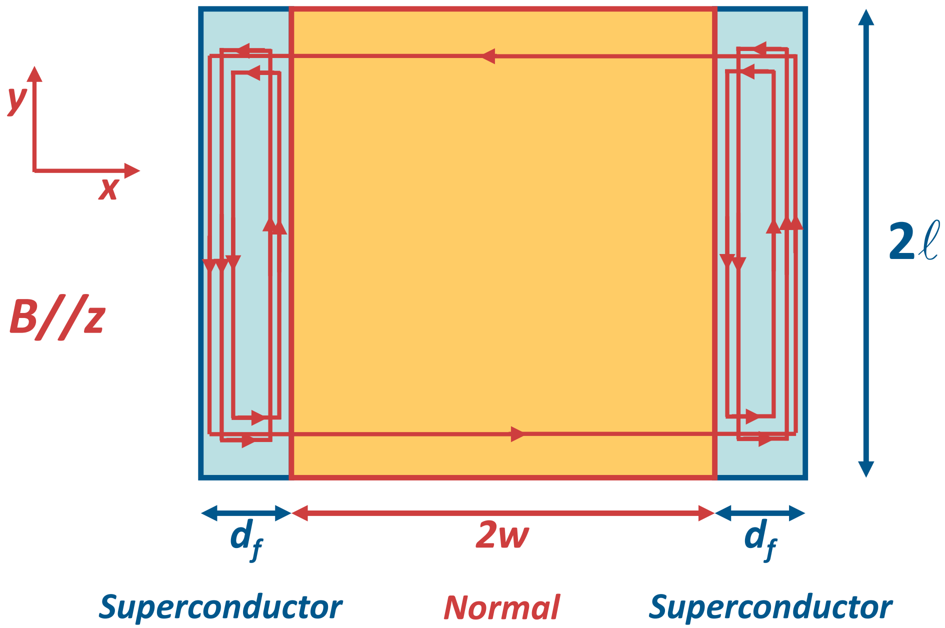


If the interfilament matrix resistivity is too low, filaments are coupled. Again

- *Losses when field is varied*
- *Flux jumps*

The solution is TWISTING !!

Multifilamentary wires, coupling and twisting



The external field is increased along z at a rate $\dot{B}$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \dot{B}$$

For the geometry of the problem $E_y = 0$

$$E_x = \mu_0 \dot{B} y$$

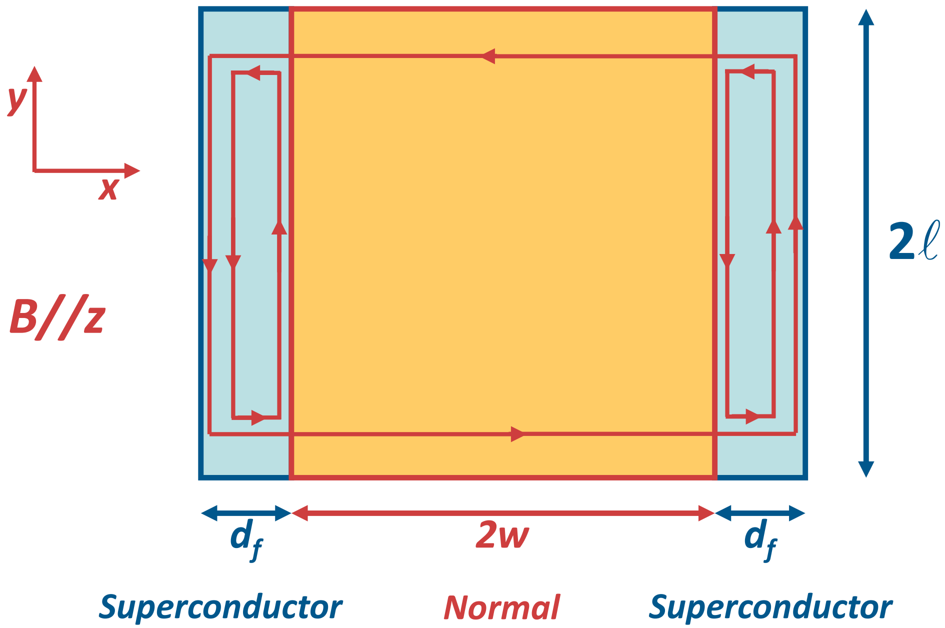
Once the electric field is known, the current density J_n in the normal metal is given by $J_n = E_x/\rho_n$

The current flowing from one superconducting slab to the other over half the conductor height is

$$I_n = \int_0^{\ell} J_n dy = \frac{\mu_0 \dot{B}}{\rho_n} \int_0^{\ell} y dy = \frac{\mu_0 \dot{B}\, \ell^2}{2\rho_n}$$

NB The current is defined per unit length in the z-direction, the slab is infinite in z

Multifilamentary wires, coupling and twisting



The current flowing from one superconducting slab to the other is

$$I_n = \frac{\mu_0 \dot{B}\,\ell^2}{2\rho_n}$$

If the two superconducting slabs are fully coupled

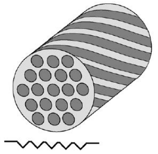
$$I_n = J_c d_f$$

This occurs for values of ℓ *beyond a certain* ℓ_c

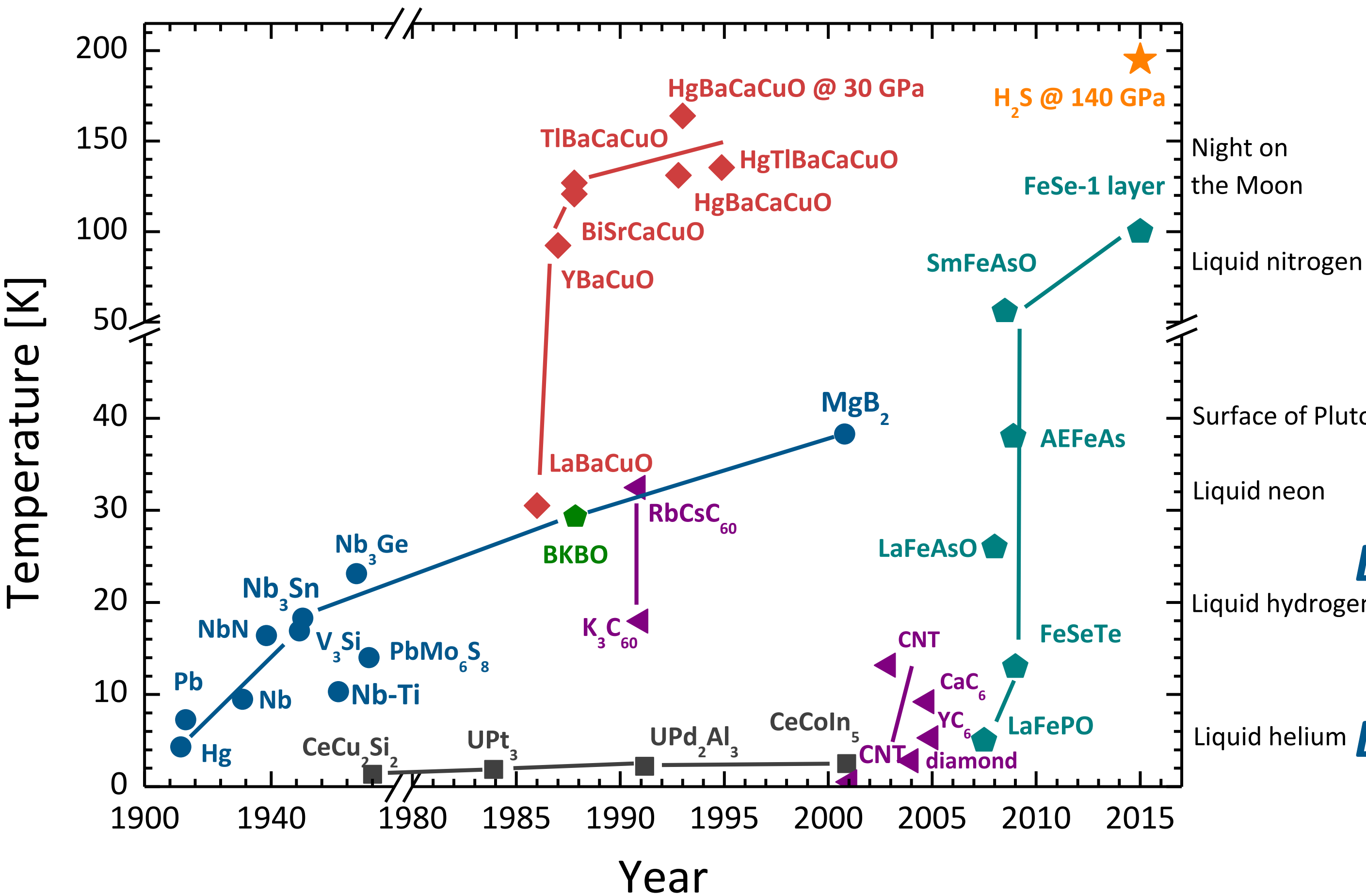
$$\boxed{\ell_c = \sqrt{\frac{2\rho_n J_c d_f}{\mu_0 \dot{B}}}}$$

If $\ell << \ell_c$ *there is no current sharing with the matrix and filaments are uncoupled*

The filament twist pitch must be smaller than ℓ_c

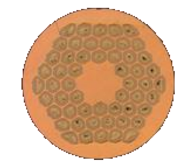


Superconductors History



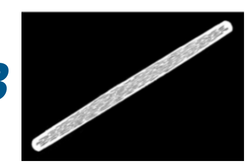
Nb-Ti

Nb_3Sn

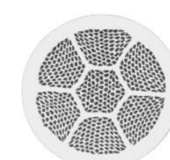


MgB_2

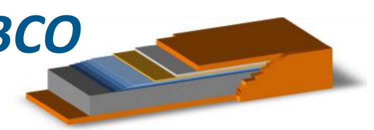
Bi2223



Bi2212



YBCO



From superconducting materials…

…to technical superconductors

1. *Superconducting ?* 10’000
2. T_c > 4.2K & B_{c2} > 10T ? 100
3. J_c > 1000 A/mm^2 ? ~10

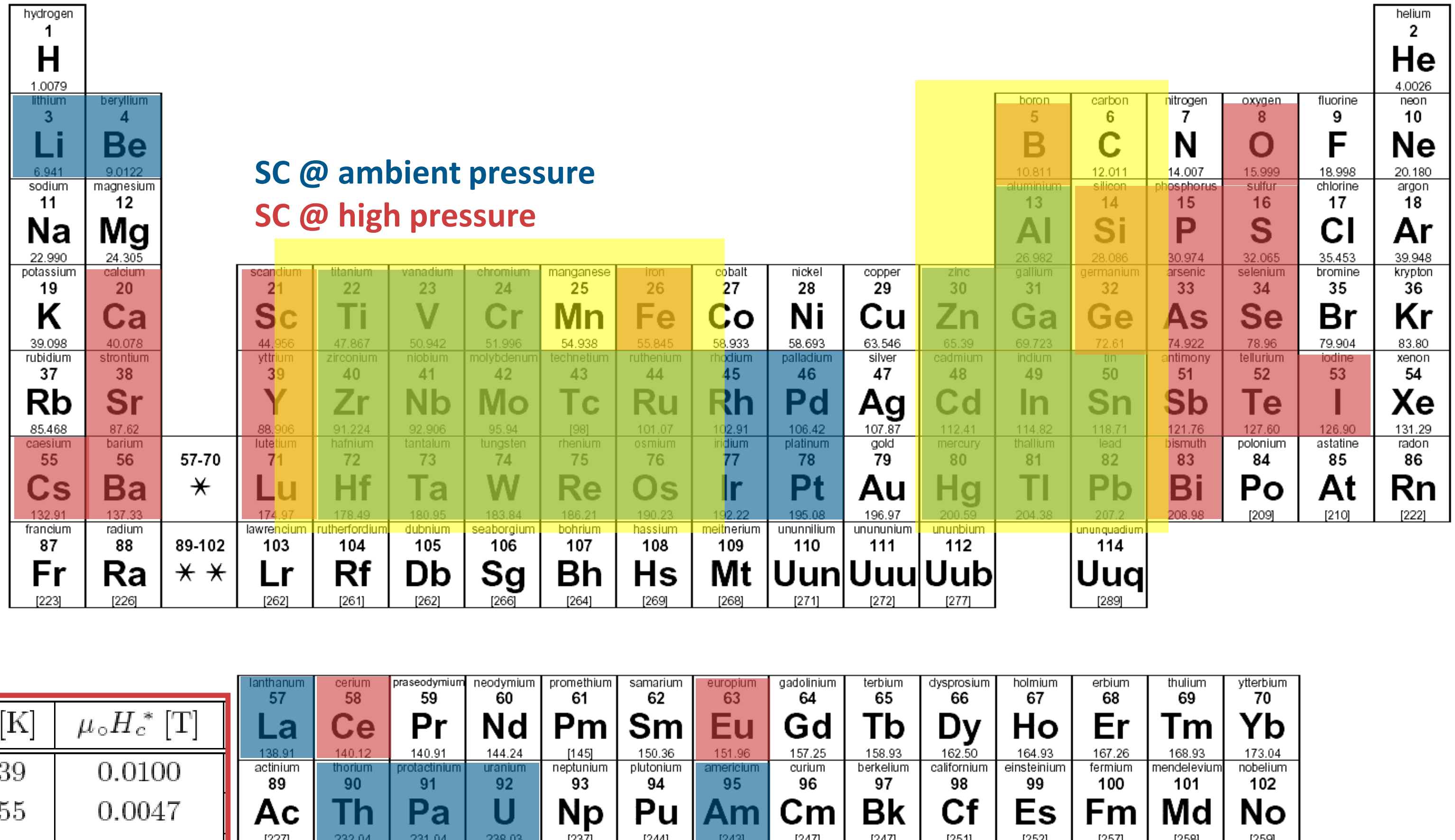
From superconducting materials…

…to technical superconductors

The six steps

1. *Discovery*
2. *Improvement of J_c performance*
3. *Co-processing with matrix metal*
4. *Multifilament form*
5. *I_c > 100 A in length > 1 km*
6. *Thermal and mechanical stabilization*

Superconducting elements



Type I	T_c [K]	$\mu_\circ H_c^*$ [T]
Ti (metals)	0.39	0.0100
Zr	0.55	0.0047
Zn	0.85	0.0054
Al	1.18	0.0105
In	3.41	0.0281
Sn	3.72	0.0305
Hg	4.15	0.0411
V	5.38	0.1403
Pb	7.19	0.0803

Type II	T_c [K]	$\mu_\circ H_{c_2}{}^*$ [T]
Nb (metals)	9.5	0.2*

The cookbook for new superconductors before 1986

from a compilation of superconducting elements, binary compounds and solid solutions

Matthias' Rules

0. Valence number per atom between 2 and 8

Favorable valance numbers: 3, 4.7, 6.9

1. Seek high symmetry

Cubic is better

2. Seek peaks in density of state

$$T_c = 1.14\omega_{ph} \exp\left(-\frac{1}{N(E_F)V_{ph}}\right)$$

BCS, PR 108 (1957) 1175

3. Stay away from oxygen

4. Stay away from magnetism

5. Stay away from insulators

B. Matthias et al., RMP 35 (1963) 1

Record T_c = 23.2 K in Nb_3Ge

Almost all superconductors discovered after 1980 do not follow these rules !!

HTS are copper oxides

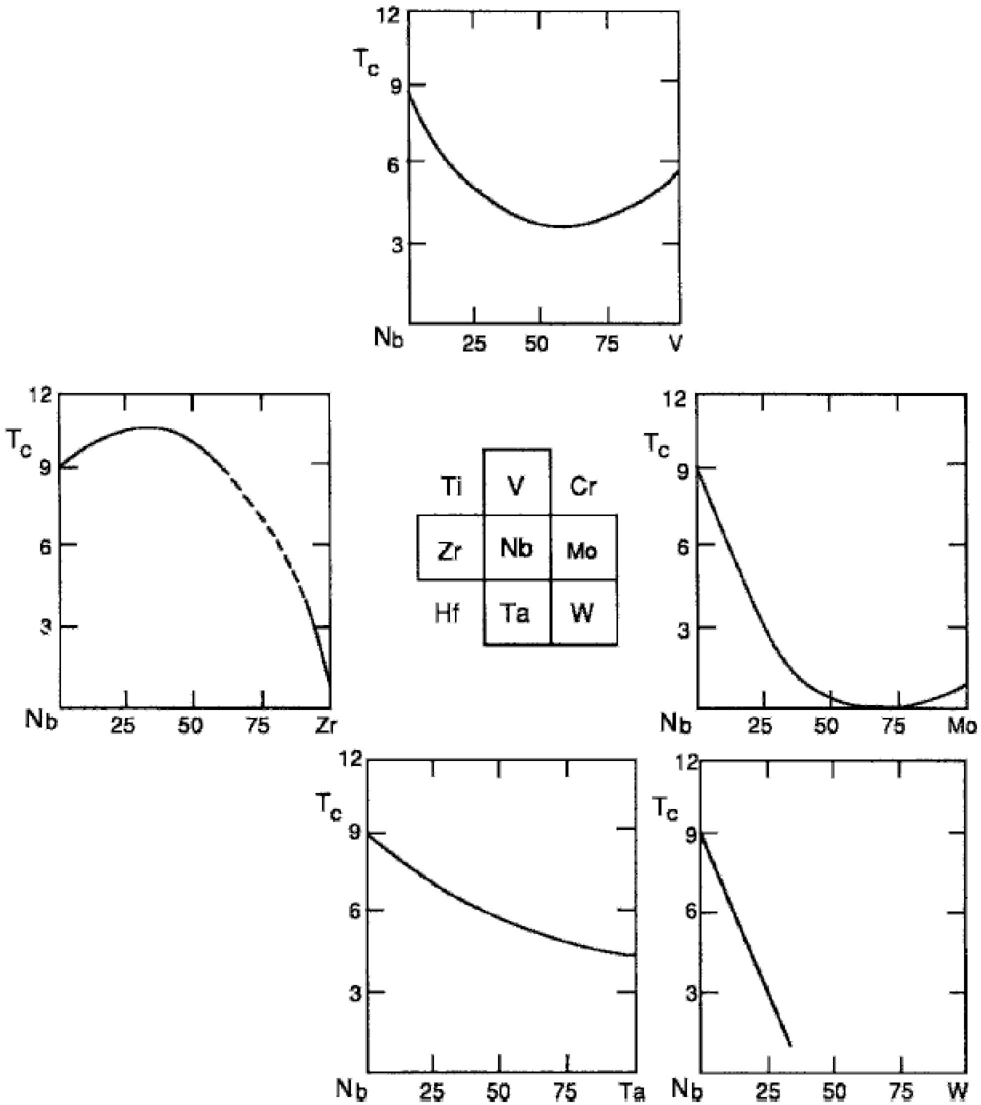
The undoped parent compounds are antiferromagnetic Mott insulators

Superconducting alloys and intermetallics

- ***An alloy is a solid solution or mixture in which atoms are randomly distributed on the lattice sites***

- ***An intermetallic compound contains definite ratios of atoms that are crystallographically ordered. There is a unit cell that replicates itself throughout the space to generate the lattice***

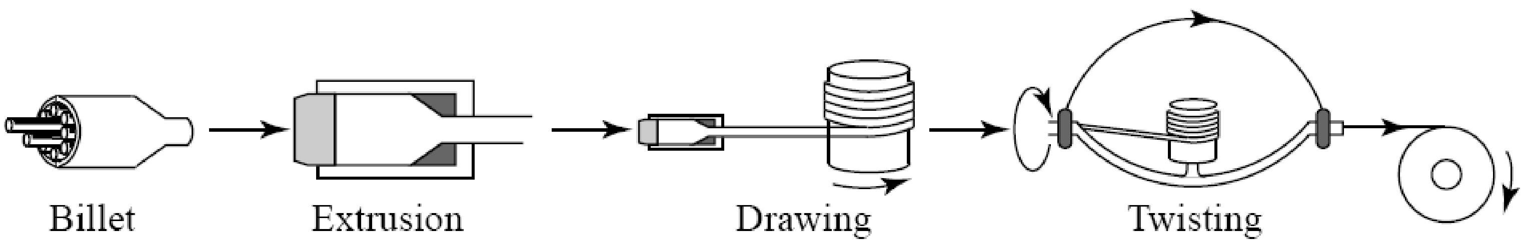
Superconducting alloys

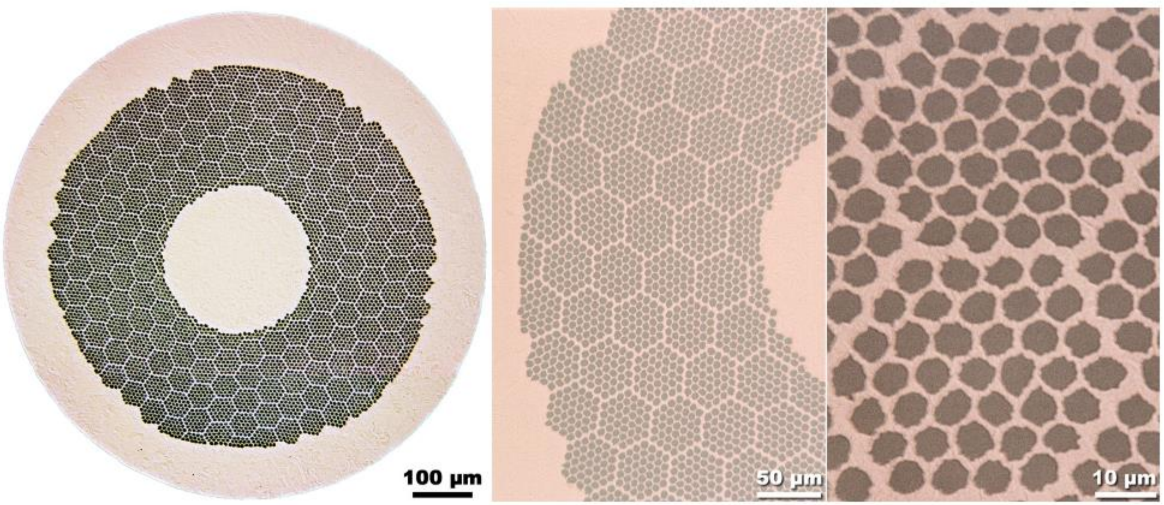


NbTi : the King of the Hill

Type II	T_c [K]	$\mu_\circ H_{c_2}{}^*$ [T]
Nb (metals)	9.5	0.2*
NbTi (alloys)	9.8	10.5†

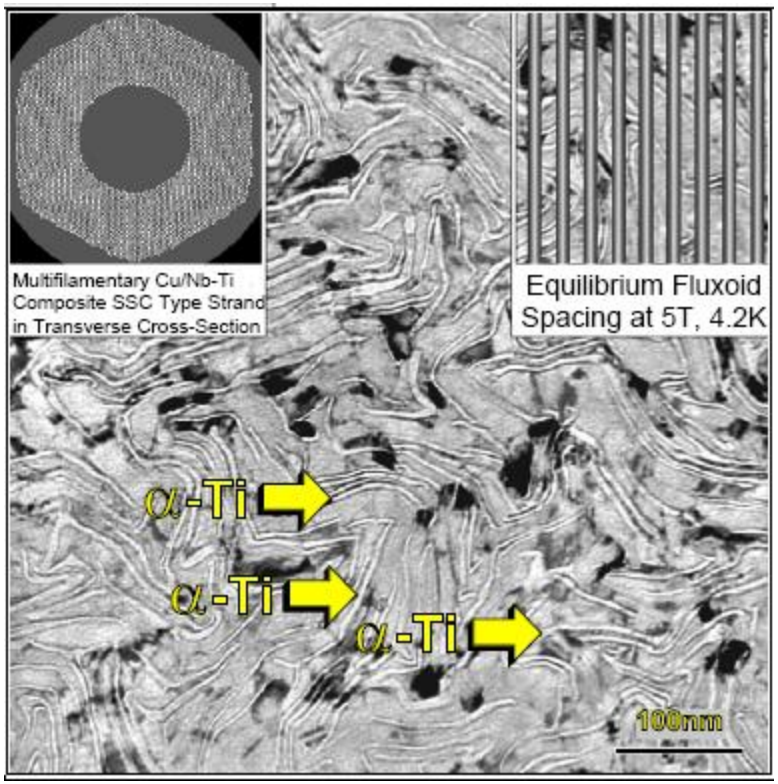
- ***Enabling technology for the large diffusion of MRI (a 4'000 M€ market!)***
- ***1200+ tonnes of NbTi in LHC***





α-Ti $\rightarrow$ hcp
β-Ti $\rightarrow$ bcc

Nb47wt%Ti : How to get high J_c



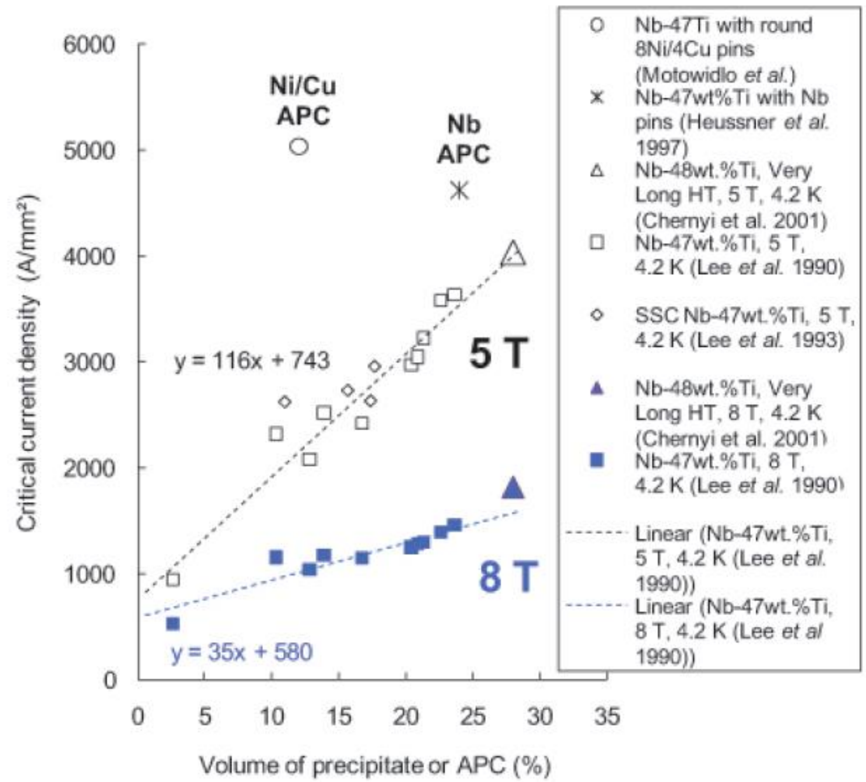
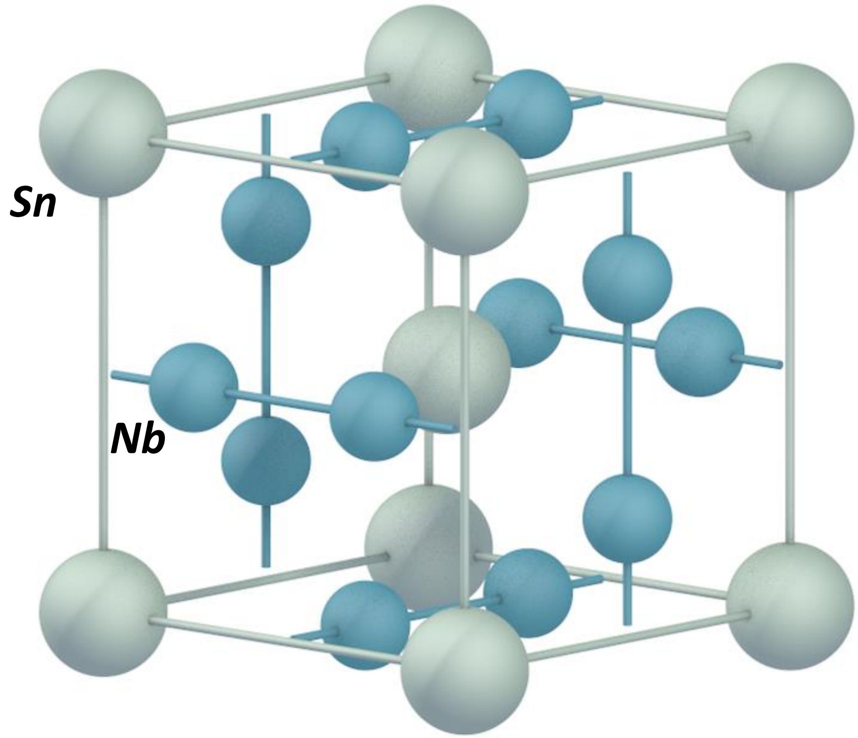


FIGURE 11.15: TEM image of the microstructure (transverse cross-section) of the first 3700 A/mm^2 (5 T, 4.2 K) multifilamentary strand from a US manufacturer (OST). This previously unpublished image taken on September 5th 1986, shows the dense array of folded α-Ti ribbons (lighter contrast) that create the strong vortex pinning.

α-Ti precipitates are adjusted to the proper dimensions in order to pin vortices

Introduction to Nb_3Sn



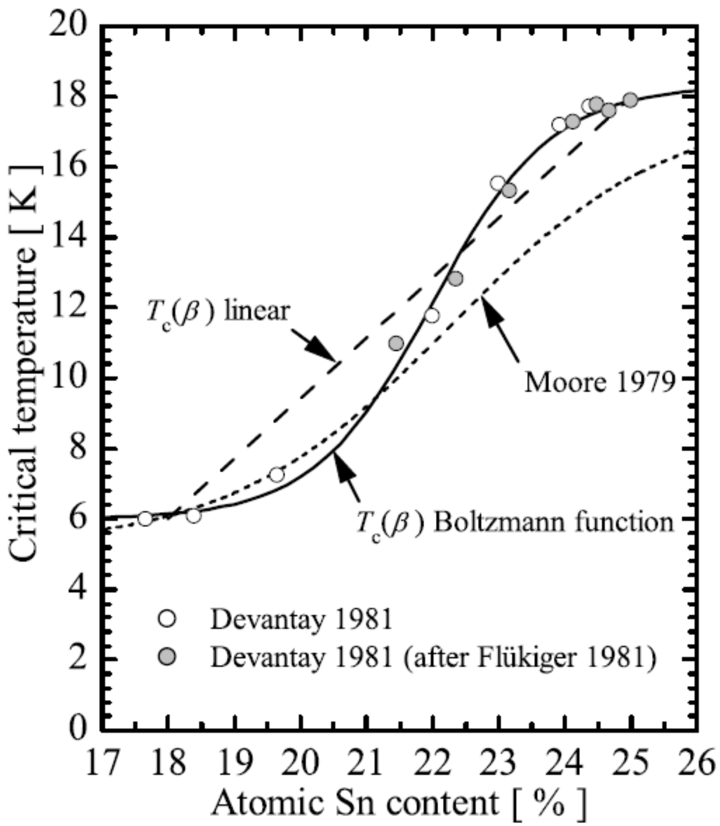
$B \backslash A_3$	Ti 4	Zr 4	V 5	Nb 5	Ta 5	Cr 6	Mo 6
Al 3			11.8	18.8			0.6
Ga 3			16.8	20.3			0.8
In 3			13.9	9.2			
Tl 3				9			
Si 4			17.1	19			1.7
Ge 4			11.2	23.2	8.0	1.2	1.8
Sn 4	5.8	0.9	7.0	18.0	8.4		
Pb 4		0.8		8.0	17		
As 5			0.2				
Sb 5	5.8		0.8	2.2	0.7		
Bi 5		3.4		4.5			
Tc 7							15.0
Re 7							15.0
Ru 8						3.4	10.6
Os 8			5.7	1.1		4.7	12.7
Rh 9			1.0	2.6	10.0	0.3	
Ir 9	5.4		1.7	3.2	6.6	0.8	9.6
Pd 10			0.08				
Pt 10	0.5		3.7	10.9	0.4		8.8
Au 11		0.9	3.2	11.5	16.0		

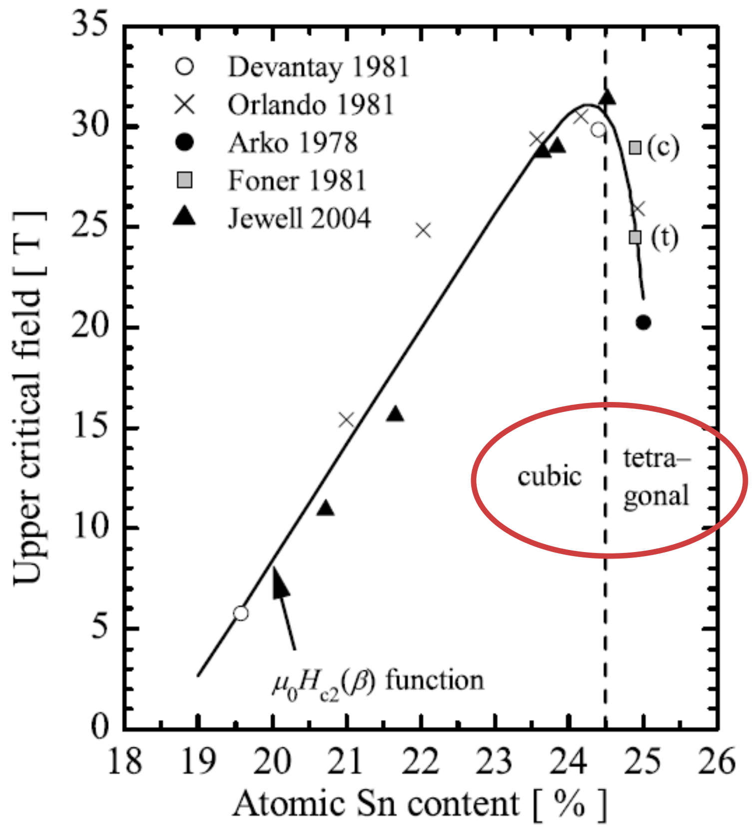
Nb_3Sn is the prototype of A15 superconductors

B.T. Matthias et al., PR 95 (1954) 1435

A15 are intermetallic compounds with A_3B formula

Nb_3Sn : the Superconductor for high fields (today)





	T_c [K]	B_{c2} [T]
Nb_3Sn	18.0	30+

$Nb_{3+x}Sn_{1-x}$ is superconducting also when deviates from stoichiometry

A. Godeke, SuST 19 (2006) R68

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Chapter 7 & 8

Iwasa
Case Studies in Superconducting Magnets
Chapter 5 Problem 5.4

Rogalla & Kes
100 Years of Superconductivity
Chapter 3 Section 7
Chapter 11 Section 2 (NbTi)
Section 3 (Nb_3Sn)