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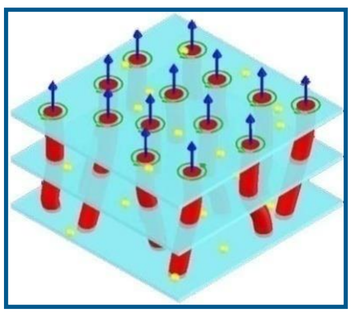
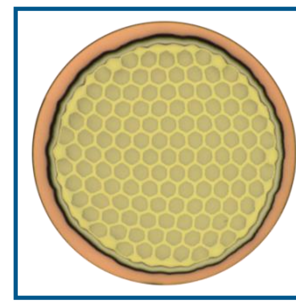
Superconductivity and its applications

Lecture 4





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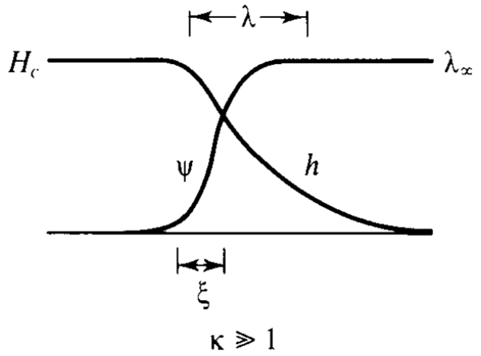
Previously, in lecture 3

Type-II superconductors: Mixed state and quantized vortices



$$\kappa > \frac{1}{\sqrt{2}} \Rightarrow \Delta E < 0$$

wall energy

- ***Magnetic flux penetrates beyond H_{c1}***
- ***Being the wall energy negative, the system prefers to maximize the walls***
- ***The entering flux is fractionated in vortices, each one carrying a flux quantum*** $\Phi_0 = \frac{hc}{2e}$

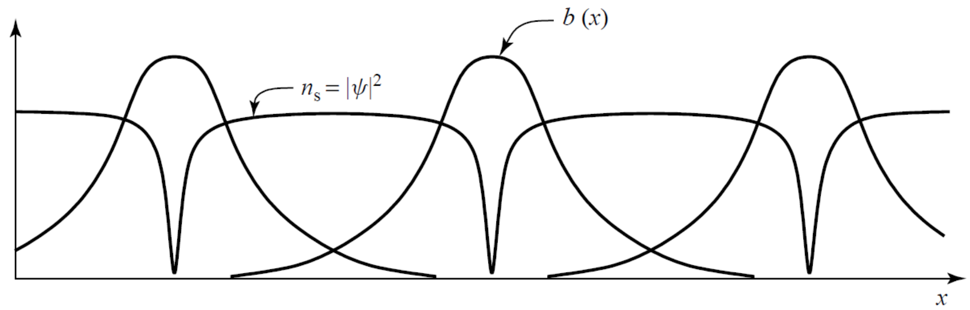
Previously, in lecture 3
From the Ginzburg-Landau equations

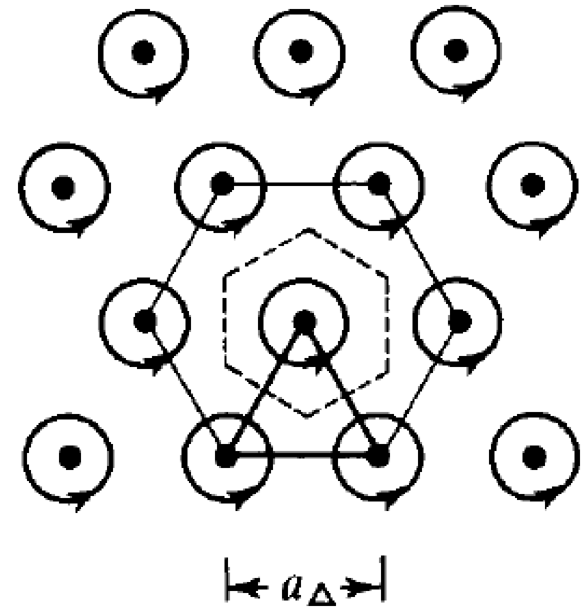
In a type-II superconductor, the critical fields are related to the characteristic lengths

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda^2} \ln\kappa = \frac{H_c}{\sqrt{2}\kappa} \ln\kappa$$

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2} = \sqrt{2}\kappa\, H_c$$

The structure of the vortex lattice



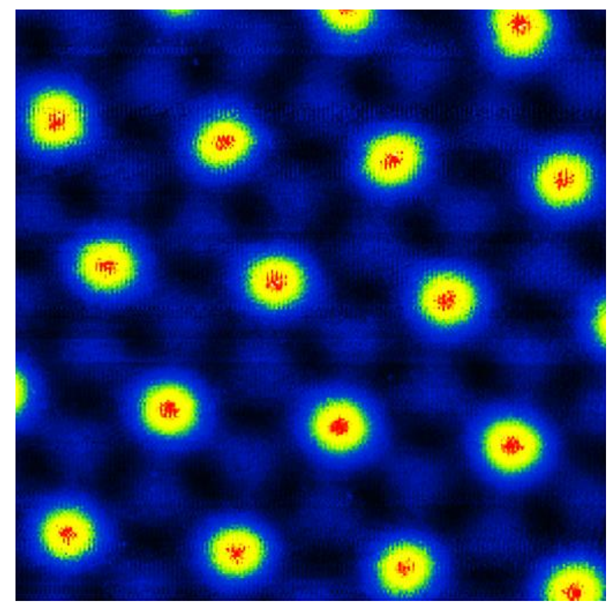


$$|\psi(r)| = \psi_\infty \tanh \frac{r}{\sqrt{2}\xi} \qquad h(r) \approx \frac{\Phi_0}{2\pi\lambda^2}\left[\ln\frac{\lambda}{r} + 0.12\right]$$

From the solution of the linearized 1st G-L equation at H_{c2}

$$\psi_L = \sum_n C_n \psi_n = \sum_n C_n \exp(inqy) \exp\left[-\frac{(x-x_n)^2}{2\xi^2}\right]$$

$$x_n = \frac{nq\Phi_0}{2\pi H} \text{ and } C_n = C_{n+\nu}$$



Interaction between vortices

In lecture 3, we found

$$\varepsilon_{1-vortex} \approx \frac{\Phi_0}{8\pi} h(0)$$

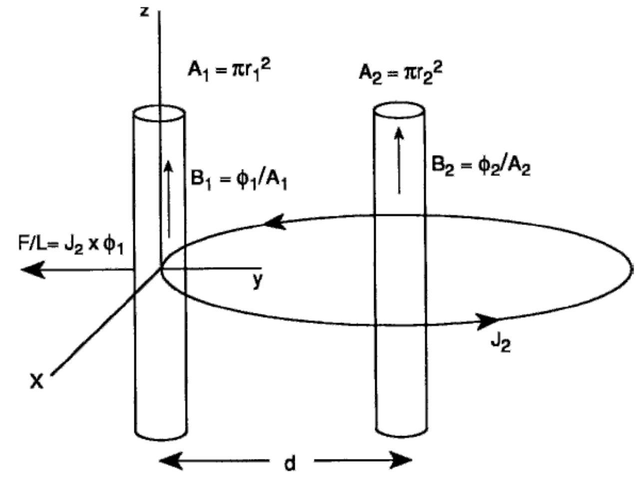
and in the case of 2 vortices

$$\varepsilon_{2-vortices} = \frac{\Phi_0}{8\pi}\left[h_1(r_1) + h_1(r_2) + h_2(r_1) + h_2(r_2)\right]$$

$$= 2\left[\frac{\Phi_0}{8\pi} h_1(r_1)\right] + \frac{\Phi_0}{4\pi} h_1(r_2)$$

$$= 2\varepsilon_{1-vortex} + \varepsilon_{interaction}$$

Interaction between vortices



The force of vortex 1 on vortex 2 is

$$f_2 = -\nabla \varepsilon_{interaction} = J_1(r_2) \times \frac{\Phi_0}{c}\hat{z}$$

The obvious generalization to an arbitrary array is

$$f = J_s \times \frac{\Phi_0}{c}\hat{z}$$

J_s is the total supercurrent due to all other vortices J_{array} + any transport current J_{ext} at the vortex core position.

Obviously, at equilibrium

$$J_{array} \times \frac{\Phi_0}{c}\hat{z} = 0$$

Vortex motion and dissipation: Flux Flow

Let's focus on the effects of a transport current J_{ext}

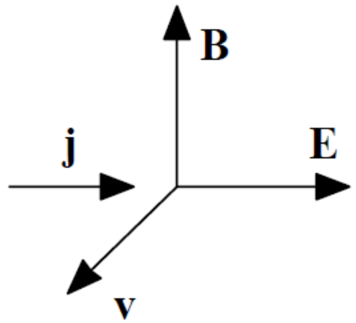
On a single vortex

$$f = J_{ext} \times \frac{\Phi_0}{c}\hat{z}$$

On the vortex lattice

$$F = \sum f = n_v f = J_{ext} \times n_v \frac{\Phi_0}{c}\hat{z} = J_{ext} \times \frac{B}{c}$$

Therefore, vortices tend to move transverse to J_{ext}. If v is their velocity



$$E = B \times \frac{v}{c}$$

DISSIPATION !!

Vortex motion and dissipation: Flux Flow

Bardeen and Stephen [PR 140 (1965) A1197] showed that the vortex velocity v is dumped by a viscous drag term

$$J_{ext}\frac{\Phi_0}{c} = \eta v_L$$

$$E = B \times \frac{v}{c}$$

And $$\rho_{ff} = \frac{E}{J} = B\frac{\Phi_0}{\eta c^2}$$

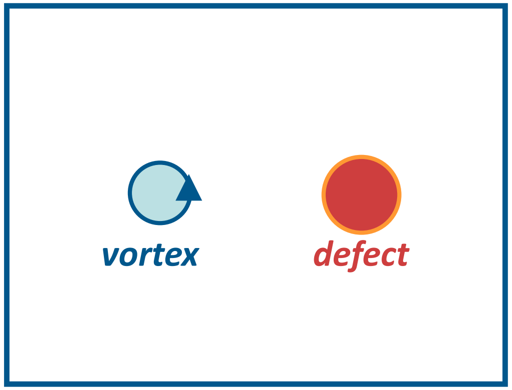
The following form is predicted for η

$$\eta = \frac{\Phi_0 B_{c2}}{\rho_n c^2}$$

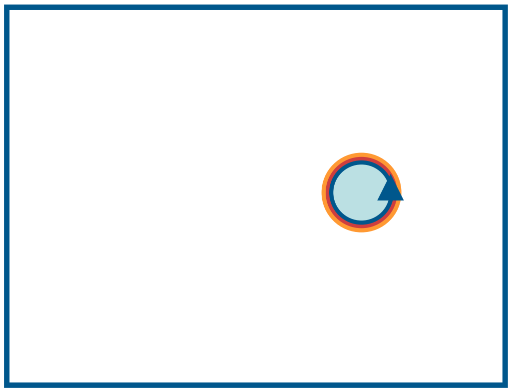
And thus $$\rho_{ff} = \rho_n \frac{B}{B_{c2}}$$

Normal fraction in the SC, occupied by the vortex cores

Vortex-defect interaction



$$\Delta G = \Delta G_{condensation}(defect) + \Delta G_{condensation}(vortex) - \Delta G_{mag}$$



$$\Delta G = \Delta G_{condensation}(defect) - \Delta G_{mag}$$

Force to extract the vortex from the defect $f_p = -\nabla U(r)$

Defects are impurities, grain boundaries and any spatial inhomogeneity, whose size is comparable with λ *and* ξ

Vertex-defect interaction

$$f = J_{ext} \times \frac{\Phi_0}{c}$$

Force exerted from J_{ext}

$$f_p = J_c \times \frac{\Phi_0}{c}$$

Pinning Force exerted from defects

J_c is the critical current density

If $f < f_p$ *then* $v = 0$ *and* $\rho = 0$

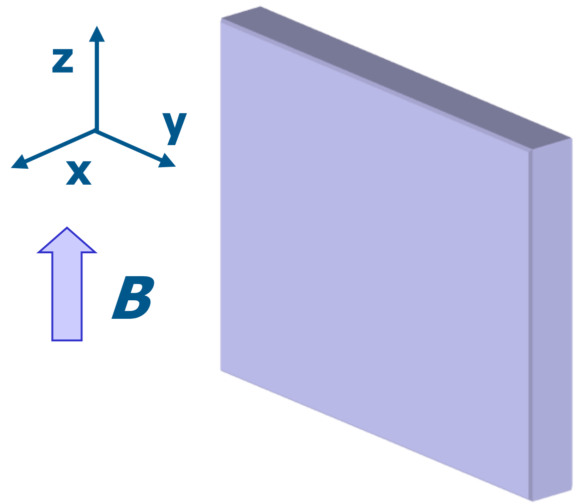
If $f > f_p$ *then* $v \neq 0$ *and* $\rho \neq 0$

Only superconductors with defects are truly superconducting ($\rho = 0$) !!

C.P. Bean, Phys. Rev. Lett. 8 (1962) 250

Critical state: the Bean model

$$\boxed{F = J \times \frac{B}{c}} \quad + \quad \boxed{\nabla \times H = \frac{4\pi}{c} J}$$



For an infinite slab in parallel field

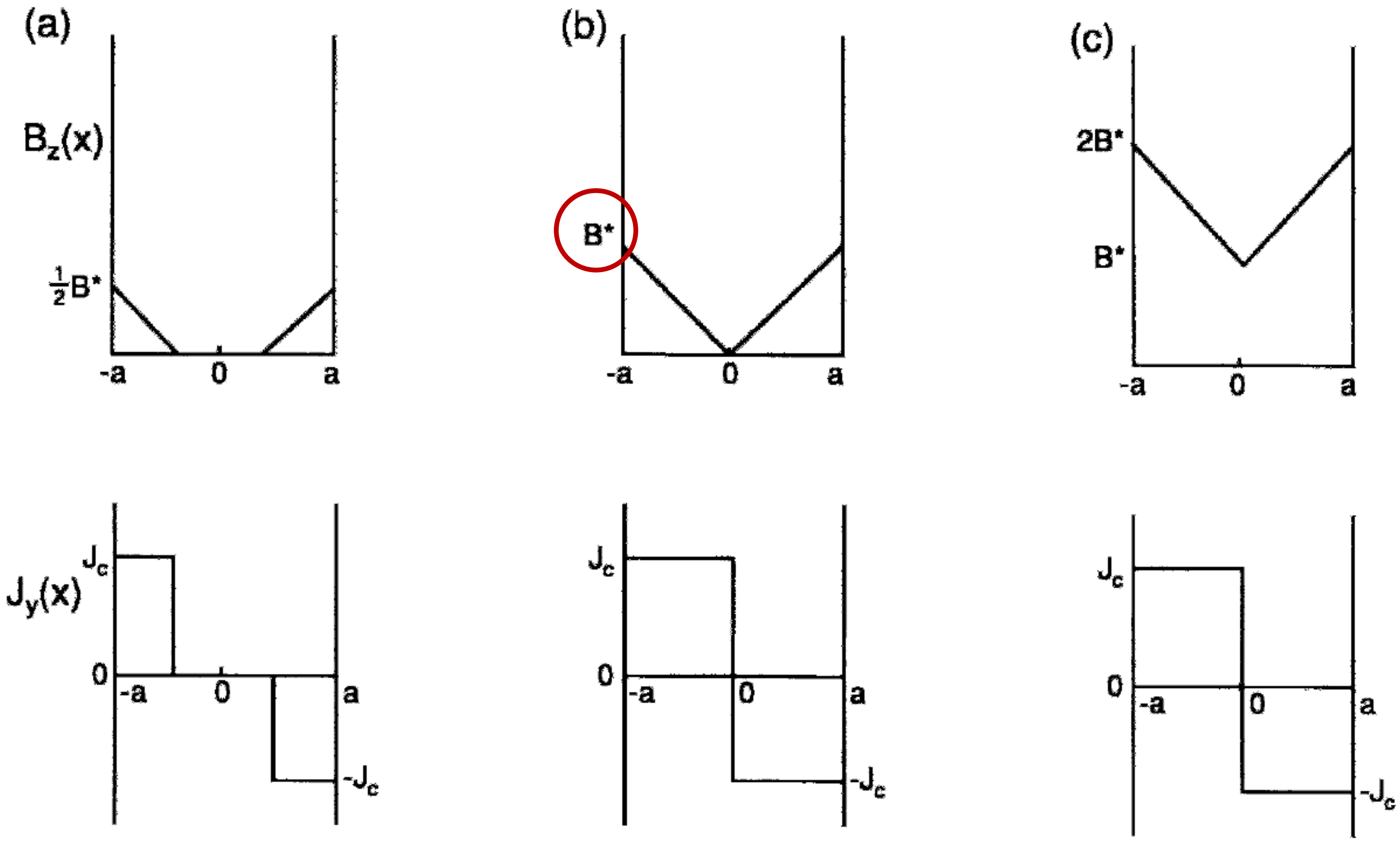
$$F = \frac{JB}{c} = \frac{1}{4\pi} B \frac{dB}{dx} \leq F_p = \frac{J_c B}{c}$$

In the critical state $\quad F = F_p \Rightarrow \frac{dB}{dx} = \frac{4\pi}{c} J_c \Rightarrow \Phi_0 \frac{dn_v}{dx} = \frac{4\pi}{c} J_c$

N.B. The critical current density J_c is different from the depairing current J_d

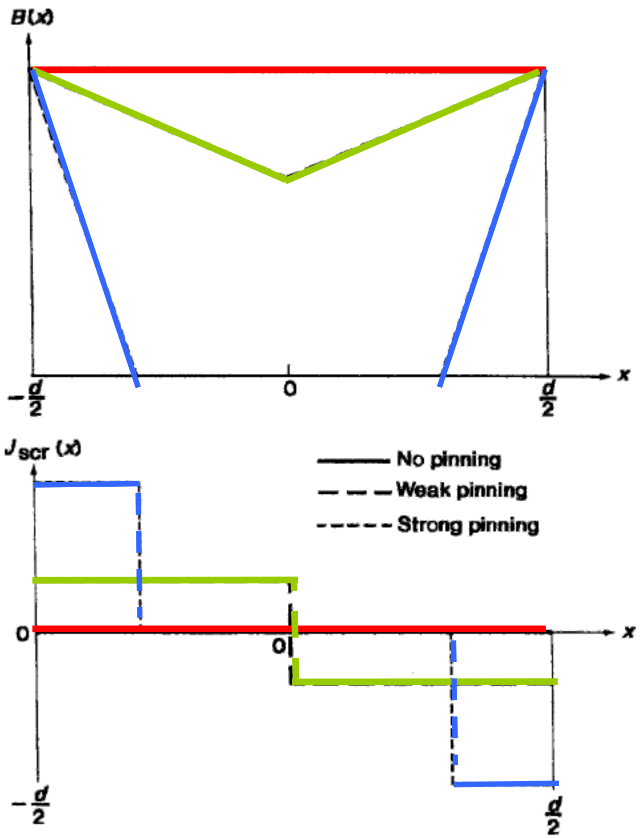
Depairing current $\quad \frac{1}{2} n_s m^* v_d^2 = \frac{2\pi}{c^2} \lambda^2 J_d^2 = \frac{H_c^2}{8\pi} \Rightarrow J_d = c \frac{H_c}{4\pi\lambda}$

Critical state: the Bean model



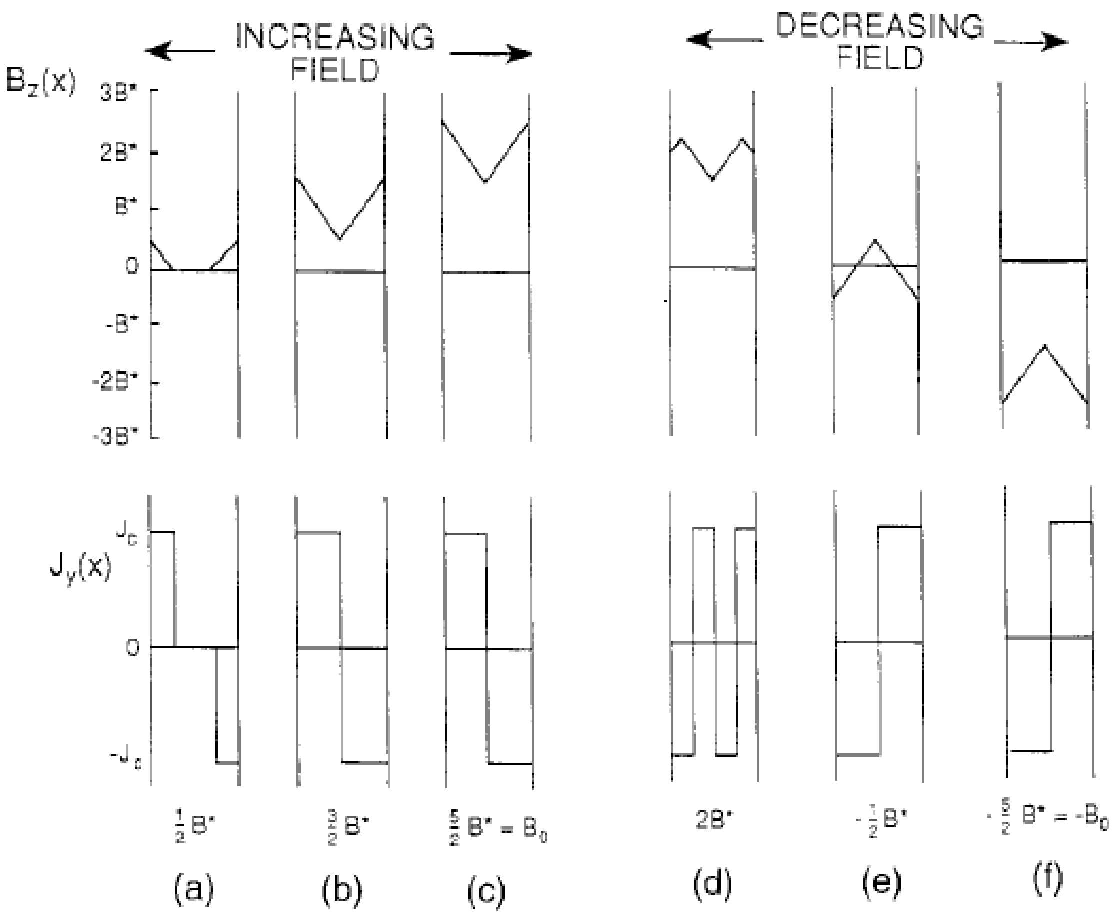
$$B^* = \frac{4\pi}{c} J_c a$$

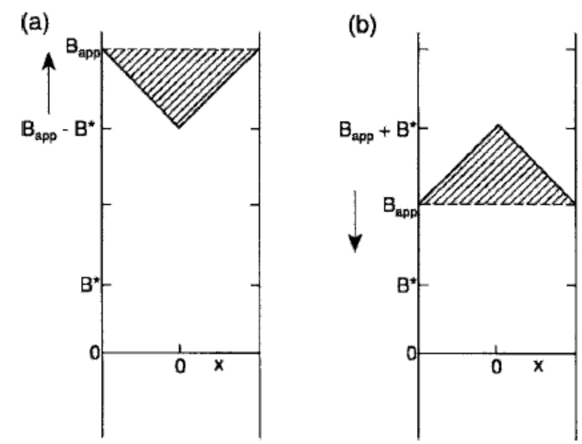
Critical state: Pinning strength



The critical current density J_c depends on the type, size and distribution of the pinning centers

Critical state: Reversing field





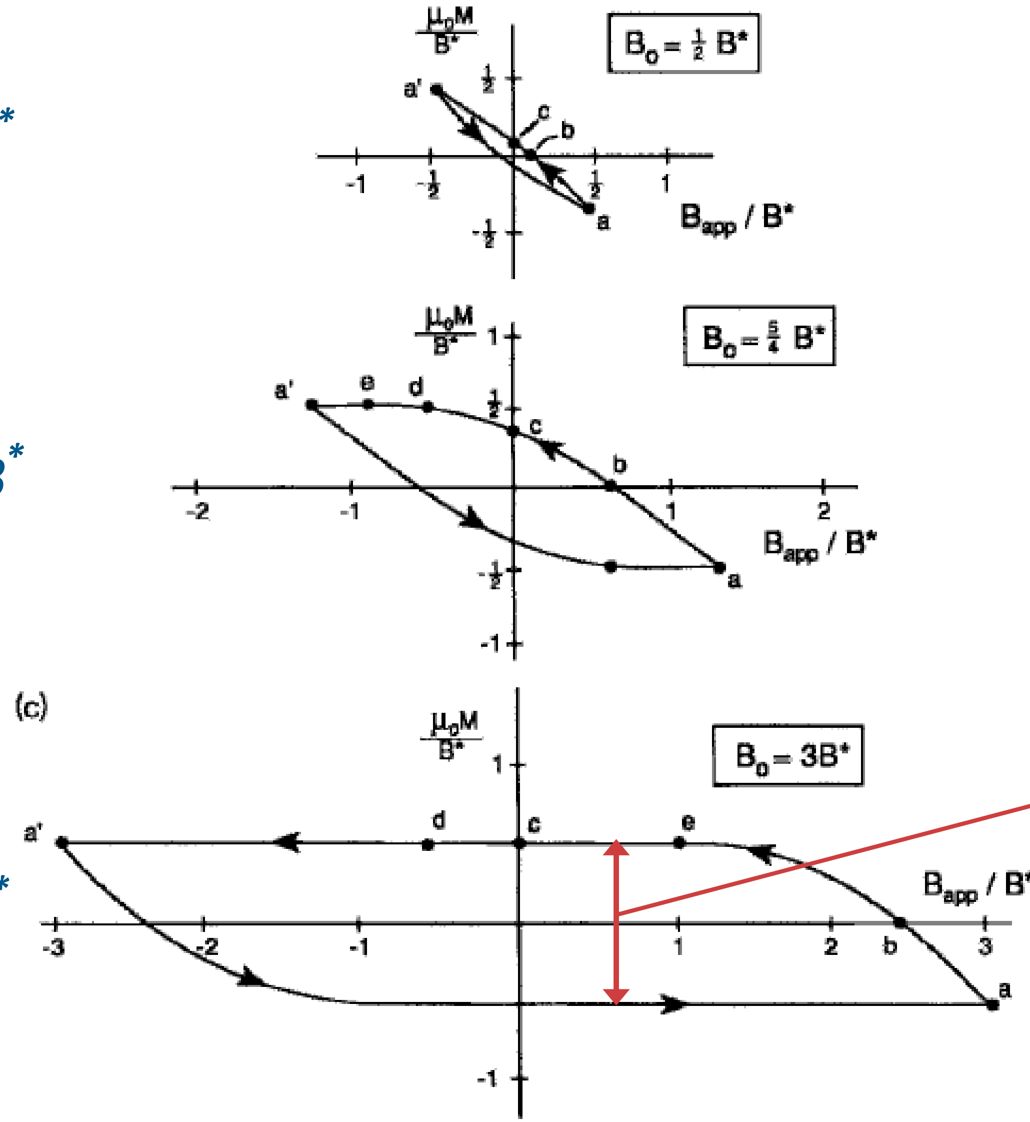
The shaded area corresponds to the sample magnetization

Critical state: Hysteresis loop

$$B_0 = \frac{1}{2} B^*$$

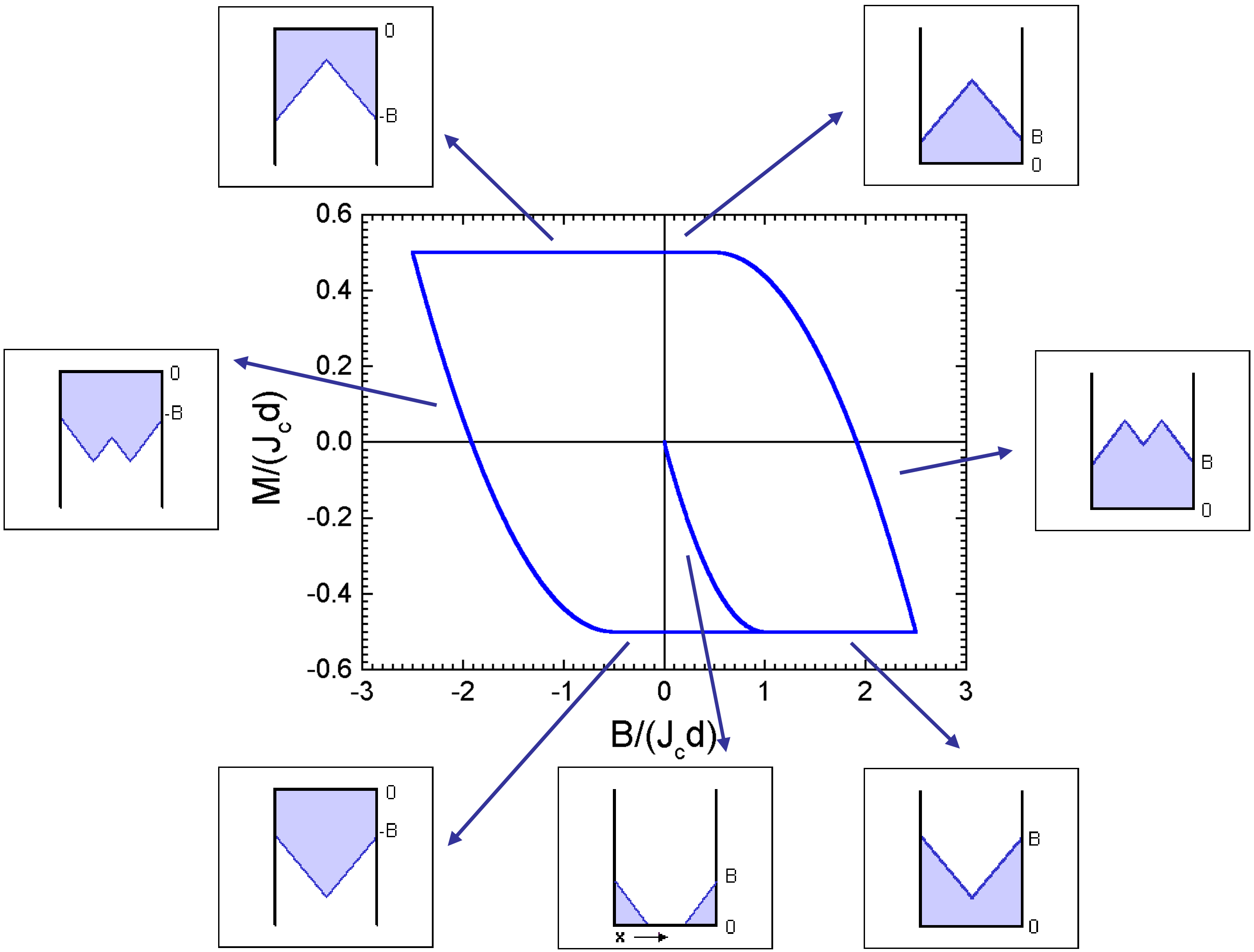
$$B_0 = \frac{5}{4} B^*$$

$$B_0 = 3B^*$$



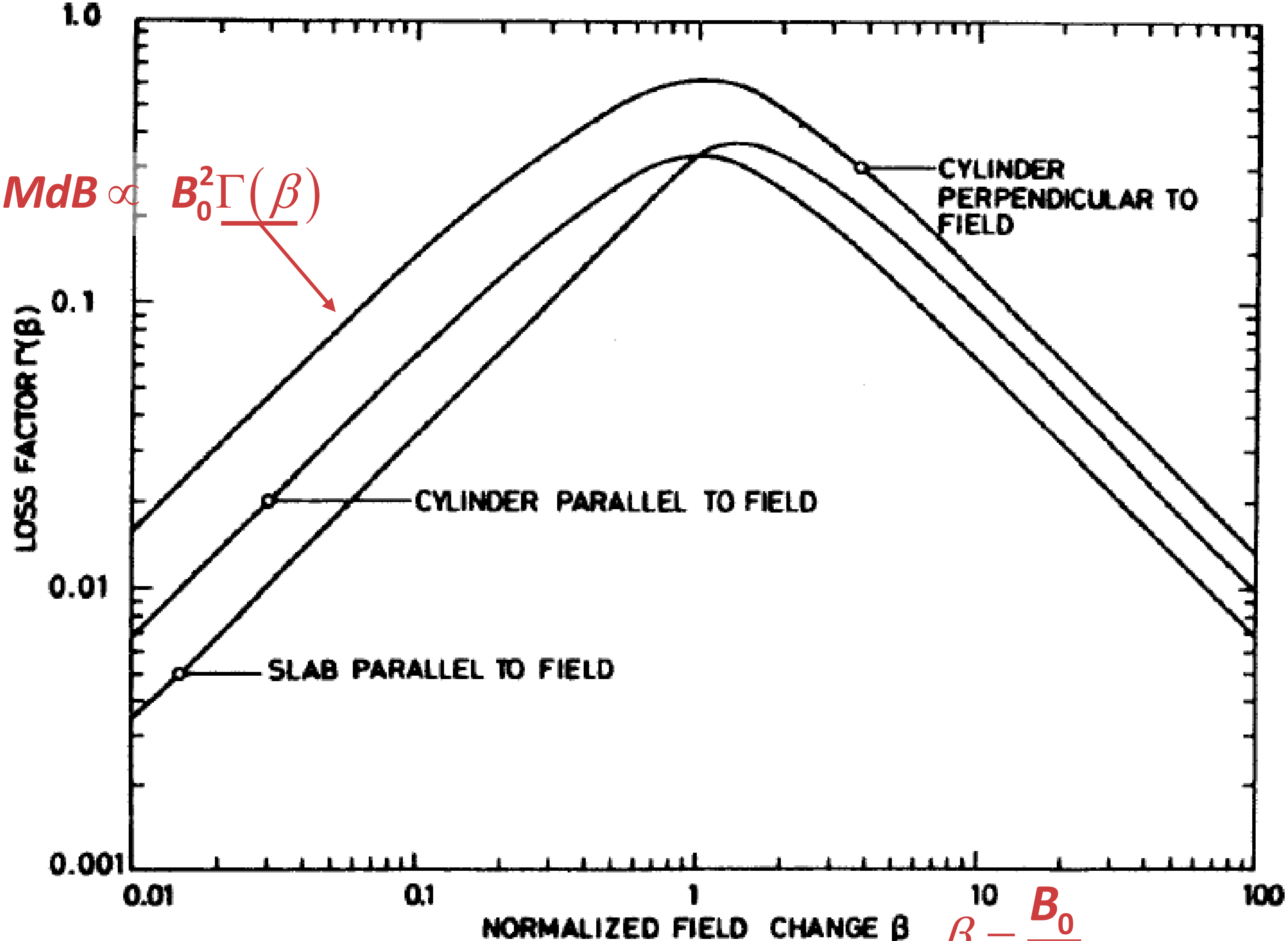
$$\Delta M = J_c a$$

Critical state: Hysteresis loop



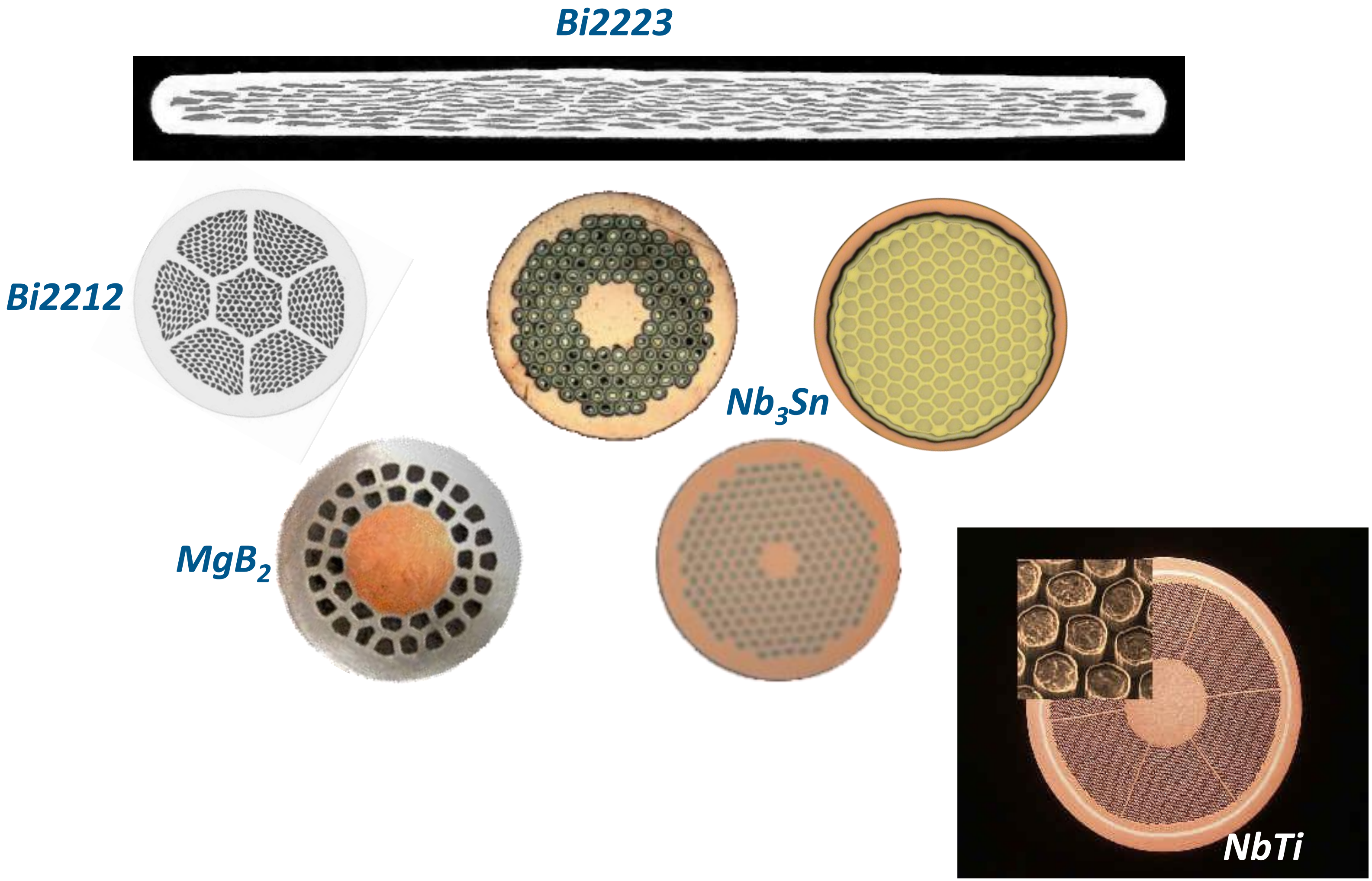
Critical state: Hysteresis and losses

$$Q = \oint M dB \propto B_0^2 \Gamma(\beta)$$

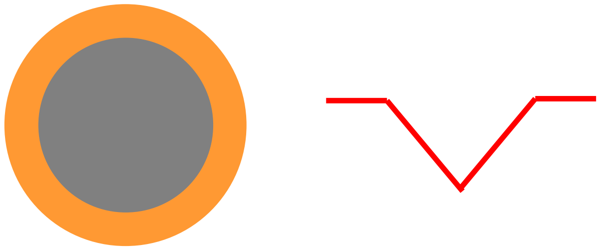


$$\beta = \frac{B_0}{B^*}$$

Superconducting wires are multifilamentary. WHY ?

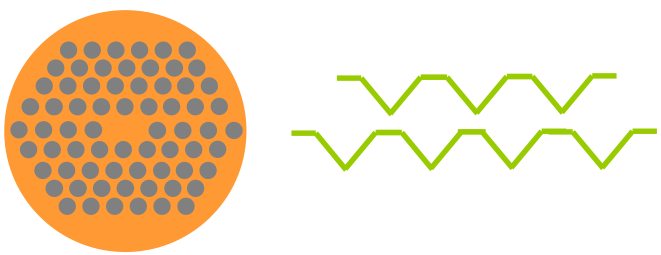


Superconducting wires are multifilamentary. WHY ?



$$\Delta M \propto J_c d$$

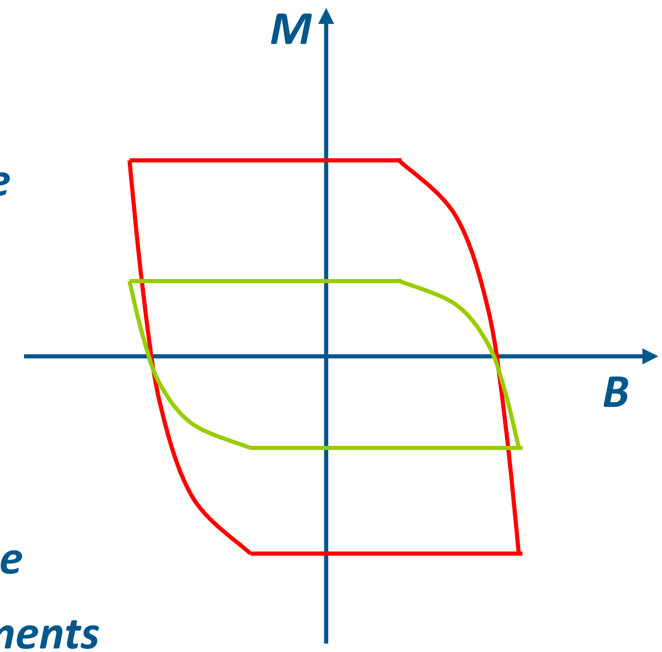
d is the filament size



$$\Delta M \propto nJ_c d$$

d is the filament size

n is the number of filaments

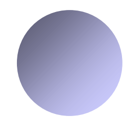


With the subdivision of the superconducting layer in filaments, hysteretic losses are reduced but the critical current density J_c is unchanged

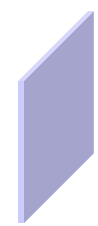
Extracting J_c from magnetization

Knowing that $M = \frac{1}{2V}\int_V r \times J \, dV$ *it is possible to calculate the expression of M for a given, constant J*

Sphere $J_c = 3\frac{\Delta M}{R}$

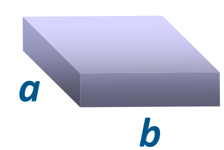


Slab in parallel field $J_c = 2\frac{\Delta M}{d}$



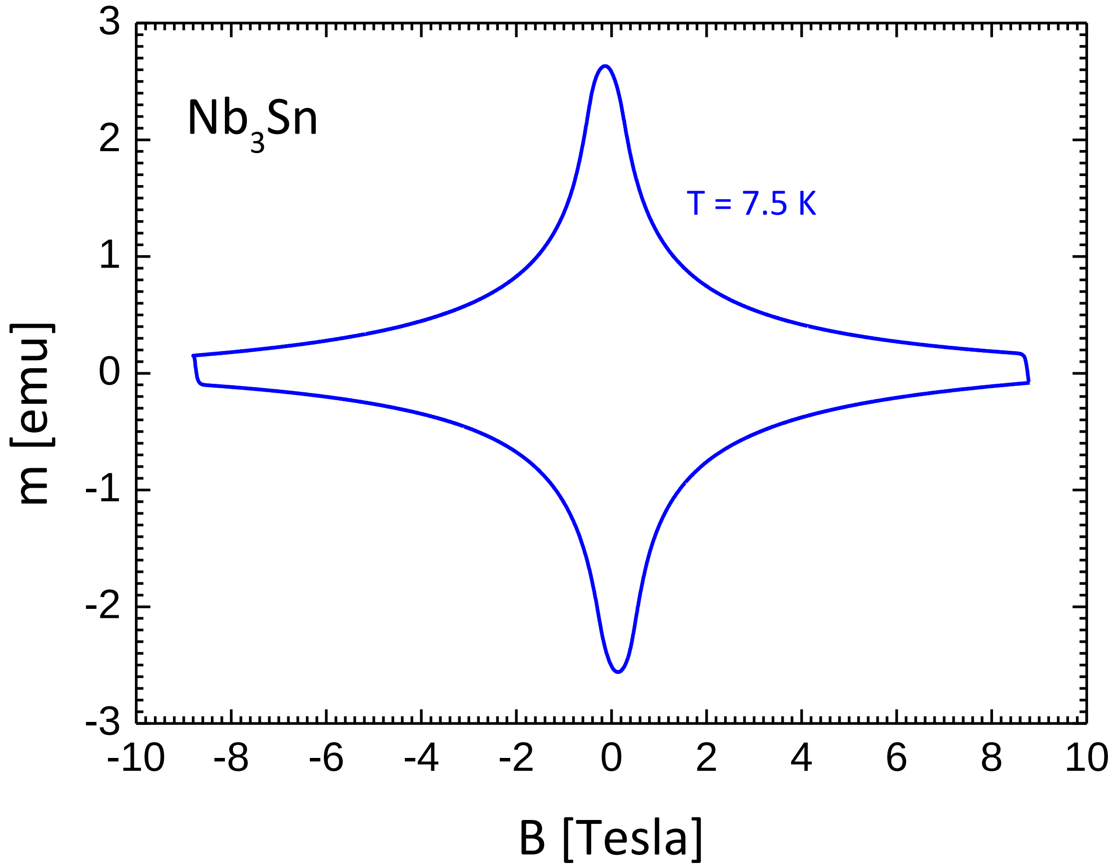
Cylinder in parallel field $J_c = \frac{3}{2}\frac{\Delta M}{R}$

Slab in perpendicular field $J_c = 3\frac{\Delta M}{b(3 - b/a)}$



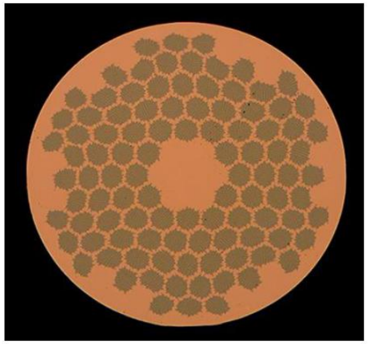
Cylinder in perpendicular field $J_c = \frac{3\pi}{8}\frac{\Delta M}{R}$

Field dependence of J_c



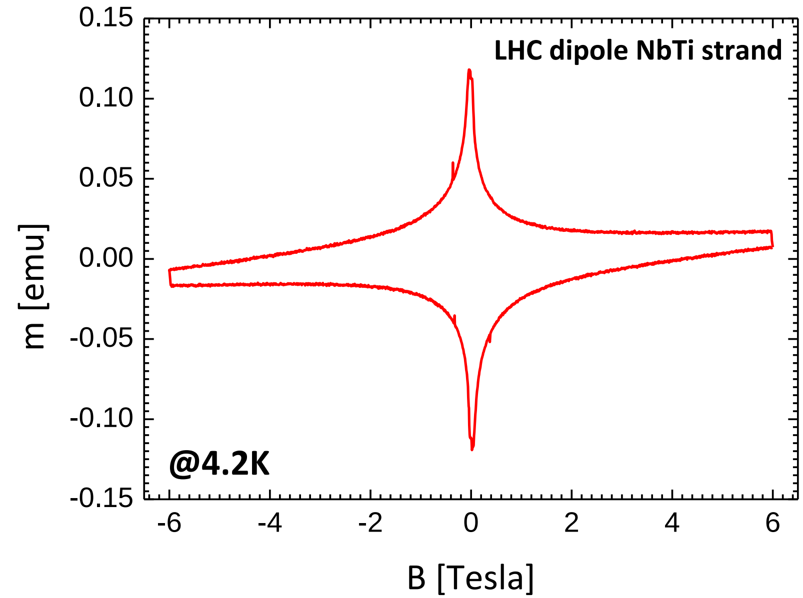
$$\Delta M = \Delta M(B) \Rightarrow J_c = J_c(B)$$

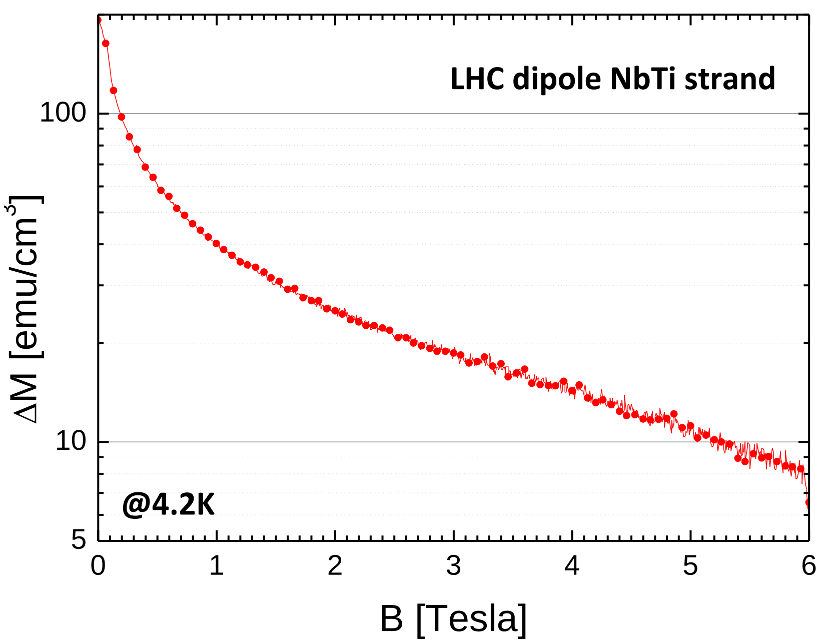
Field dependence of J_c

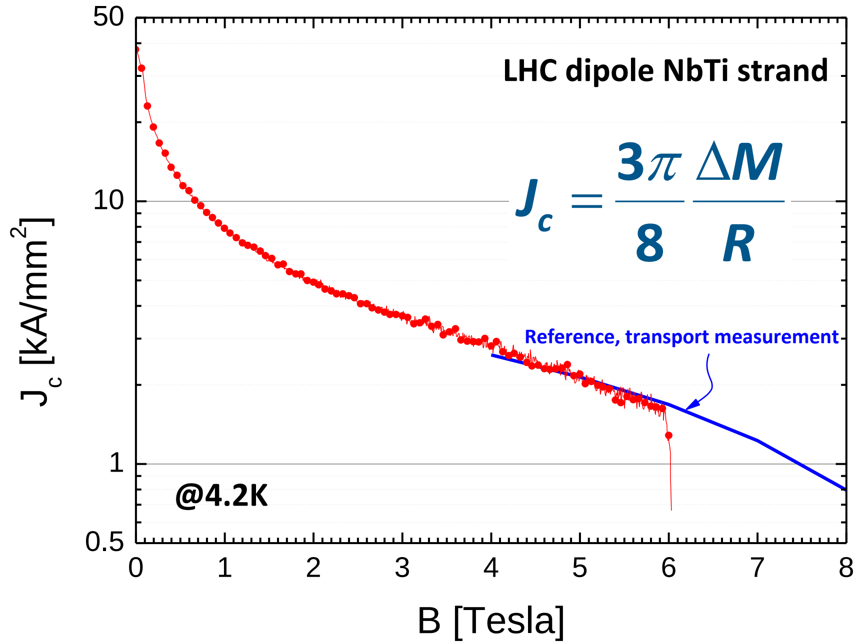


LHC dipole strand

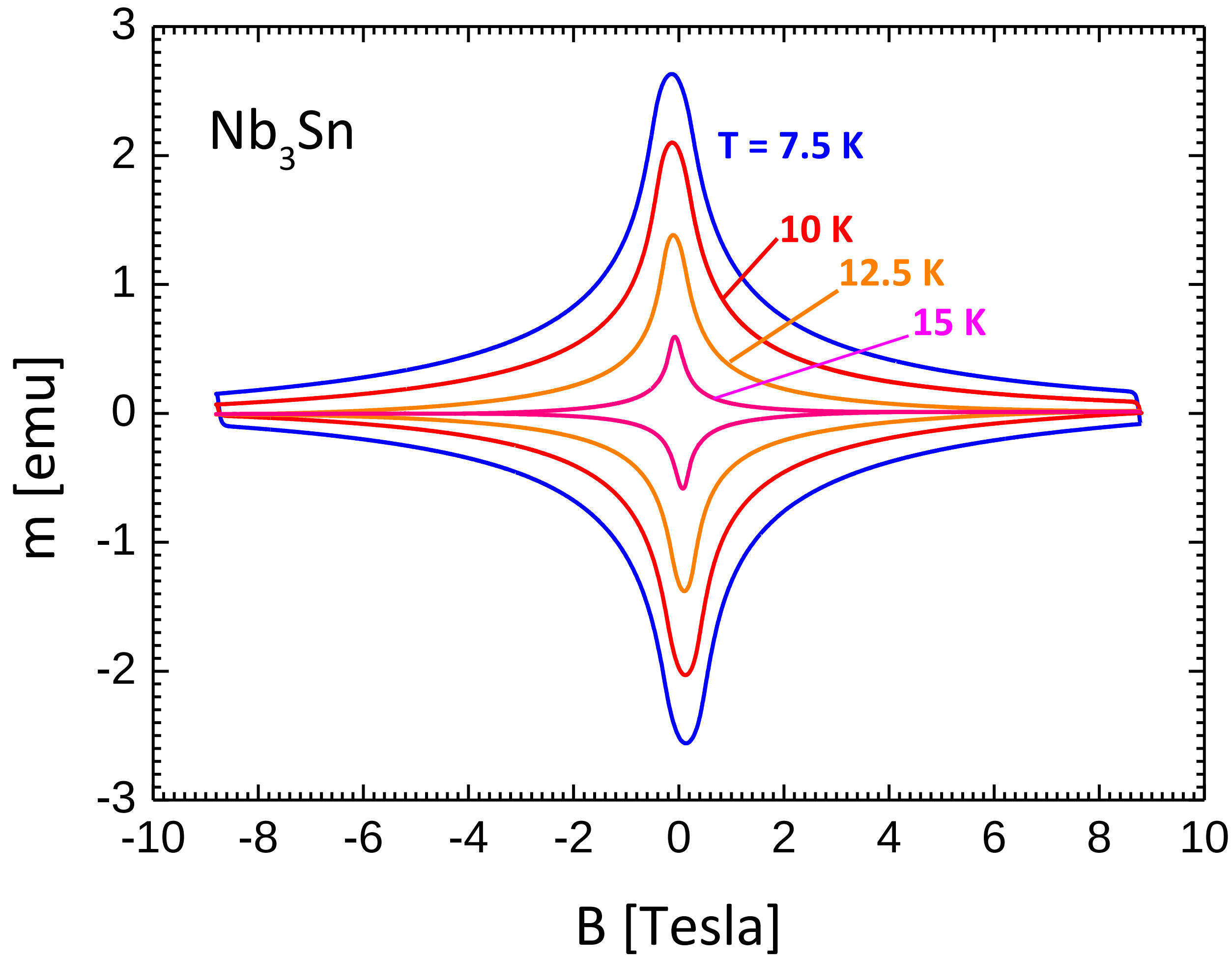
- F8670 Ø 1.065 mm
- Cu : NbTi = 1.65
- Filament diameter ≈ 7 μm
- Ic = 540 A @ 7 T; 4.2 K







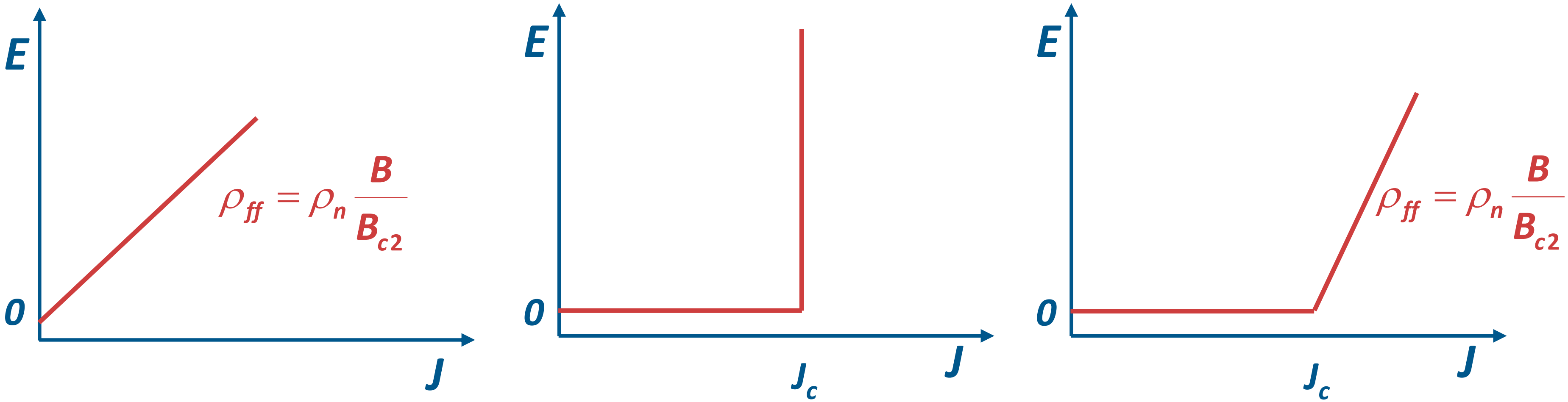
Temperature dependence of J_c



$$\Delta M = \Delta M(B,T) \;\Rightarrow\; J_c = J_c(B,T)$$

E-J relation for a type-II superconductor

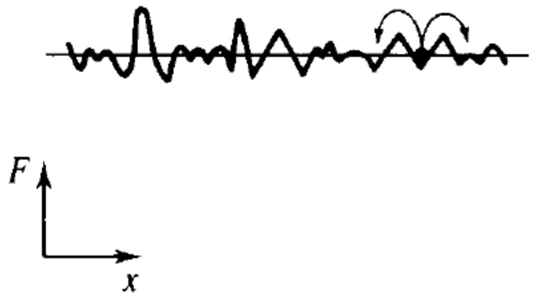
E-J relation for a superconductor in the mixed state

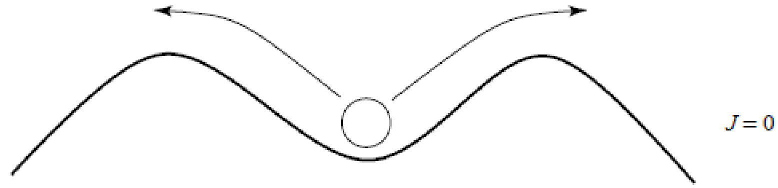


w/o pinning: flux flow | pinning: Bean Model | Bean Model + flux flow

The effect of thermal excitation was not yet included

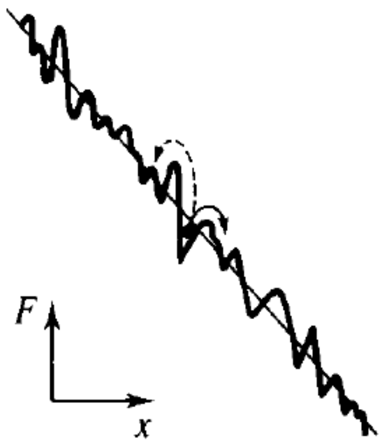
Beyond Bean: Thermally activated flux creep

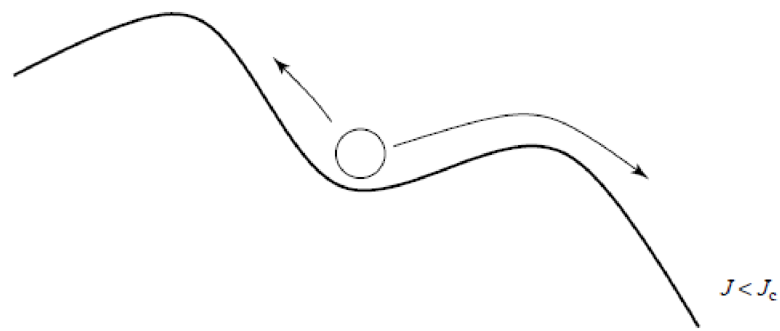




$$R = \nu_0 e^{-U/k_B T}$$

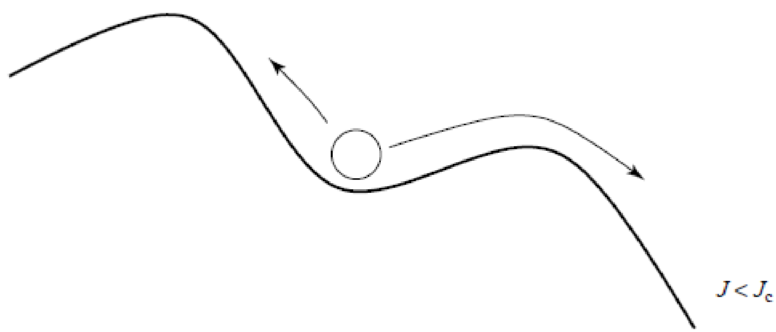
In the presence of an external current



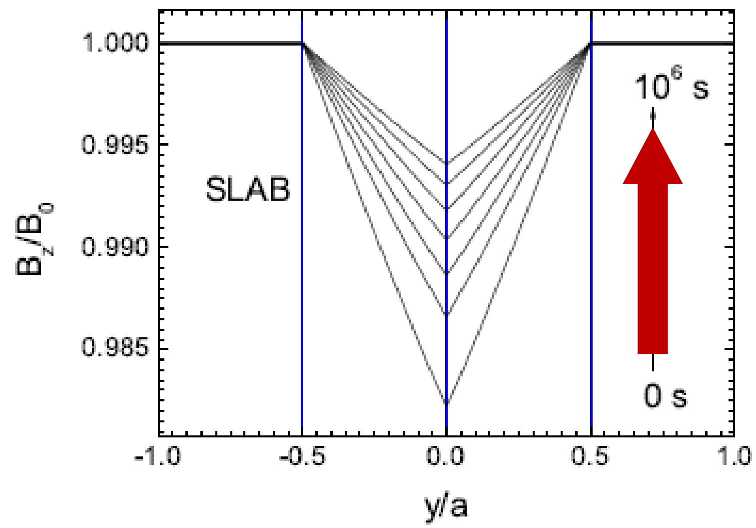


$$R = R_+ + R_- = \nu_0 e^{-U/k_B T}\left(e^{\Delta U/k_B T} - e^{-\Delta U/k_B T}\right)$$

Thermally activated flux creep



The B-profile relaxes with t



$$R = R_+ + R_- = \nu_0 e^{-U/k_BT}\left(e^{\Delta U/k_BT} - e^{-\Delta U/k_BT}\right)$$

In a first approximation

$$R \approx R_+ = \nu_0 e^{-(U-\Delta U)/k_BT} \quad \textit{where} \quad \Delta U = \Delta U(J)$$

Thermally activated flux creep

$$R \approx R_{+} = \nu_0 e^{-[U-\Delta U(J)]/k_B T} \Rightarrow \quad v = v_0 \exp\left[-\frac{U(J)}{k_B T}\right]$$

In the infinite slab – parallel field geometry

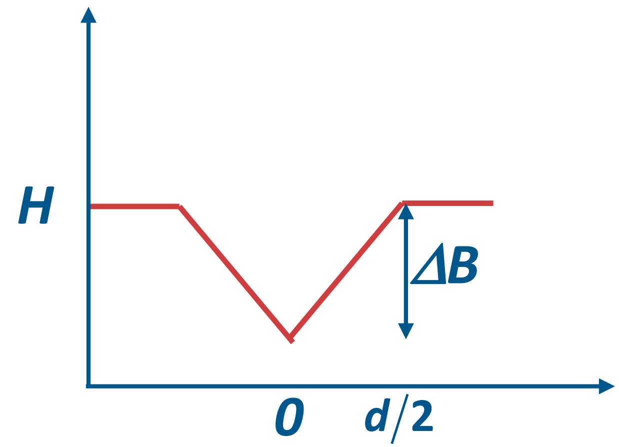
$$\boxed{\frac{\partial E}{\partial x} = \frac{\partial B}{\partial t}} + \boxed{E = vB} \Rightarrow \quad \frac{\partial B}{\partial t} = \frac{\partial}{\partial x}(vB)$$

$$\frac{\partial B}{\partial x} = -\mu_0 J \quad \text{with} \quad B\|z \quad \text{and} \quad J\|y \Rightarrow \quad \frac{\partial J}{\partial t} = -\frac{1}{\mu_0}\frac{\partial^2}{\partial x^2}(vB)$$

$$\frac{\partial J}{\partial t} = -\frac{1}{\mu_0}\frac{\partial^2}{\partial x^2}\left[v_0 e^{-\frac{U(J)}{k_B T}} B\right]$$

Thermally activated flux creep: Anderson-Kim model

$$\iint \frac{\partial J}{\partial t} = \iint \frac{1}{\mu_0} \frac{\partial^2}{\partial x^2} \left[v_0 e^{-\frac{U(J)}{k_B T}} B \right]$$



1) ΔB << H

2) J is constant in space

$$\frac{\partial J}{\partial t} = -\frac{8 v_0 H}{\mu_0 d^2} \exp\left[-\frac{U(J)}{k_B T} \right]$$

In the Anderson-Kim model $U = U_0\left(1 - \frac{J}{J_0}\right)$ ***where*** $U_0 = U_0(B,T)$

The solution for J is $J(t) = J_0\left[1 - \frac{k_B T}{U_0} \ln\left(\frac{t}{t_0}\right)\right]$

P.W. Anderson and Y.B. Kim, RMP 36 (1964) 39

Thermally activated flux creep: Anderson-Kim model

The current density J and the magnetization M both relax with the logarithm of time

$$M \propto J$$

$$J(t) = J_0\left[1 - \frac{k_B T}{U_0} ln\left(\frac{t}{t_0}\right)\right]$$

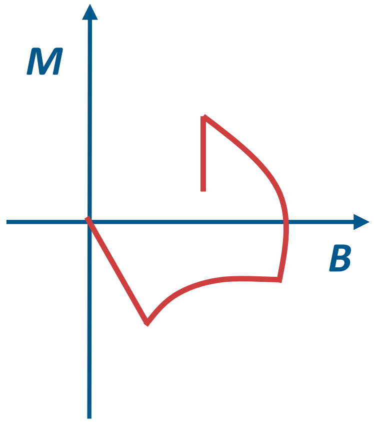
The relaxation rate S is directly proportional to T and inversely proportional to U_0

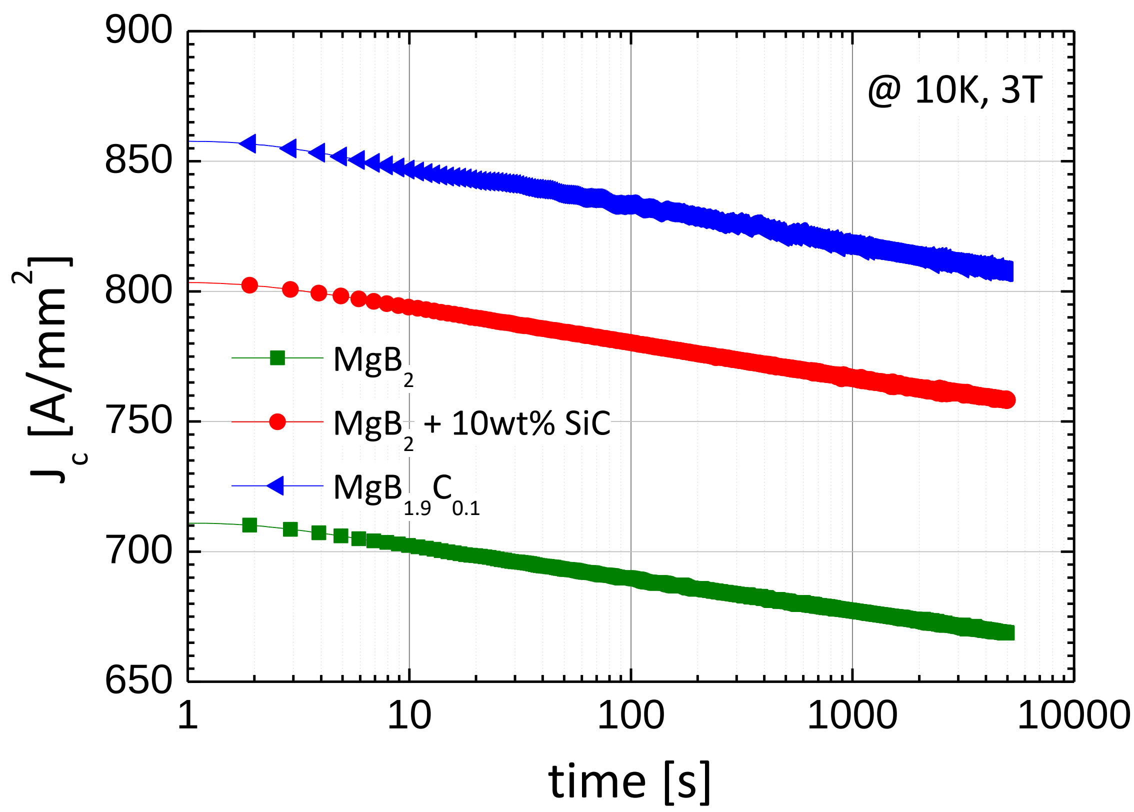
$$S = -\frac{1}{M_0}\frac{dM}{d\ln t} = \frac{k_B T}{U_0}$$

Measuring S as a function of B and T, we have an experimental access to the pinning energy

$$U_0 = U_0(B,T)$$

Magnetic relaxation experiments





Typical values of the activation energy U_0 span in the range 10-100 meV

(~100-1000 K)

Bibliography

Tinkham
Introduction to Superconductivity
Chapter 5

Fosshein & Sudbø
Superconductivity: Physics and Applications
Chapter 8