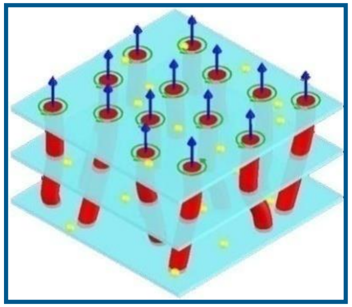
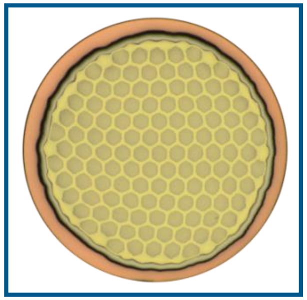
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Superconductivity and its applications

Lecture 2





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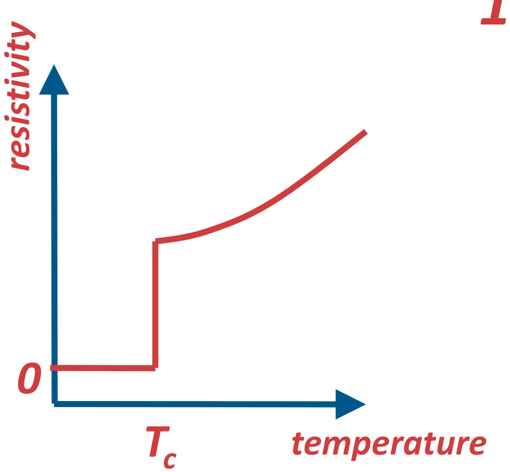
Previously, in lecture 1

If it is a superconductor, then…

1



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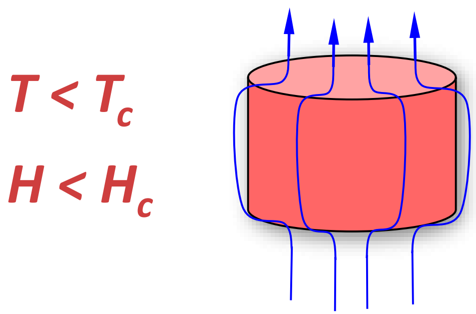

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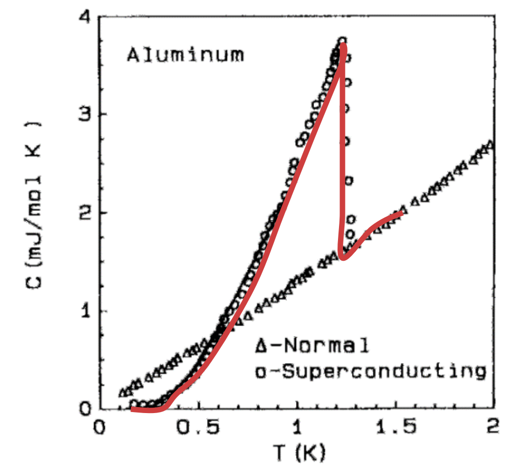
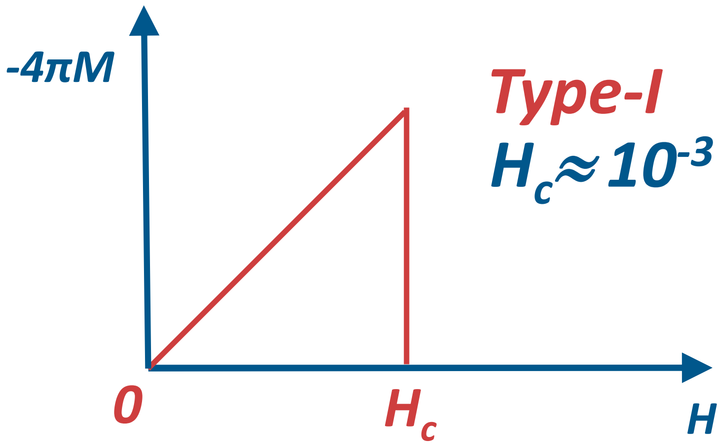


Figure 4.5 Specific heat jump in superconducting Al compared with the normal-state specific heat (Phillips, 1959; see Crow and Ong, 1990, p. 225).

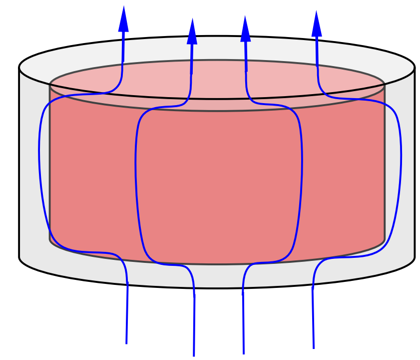
Previously, in lecture 1

Type-I and Type-II superconductors

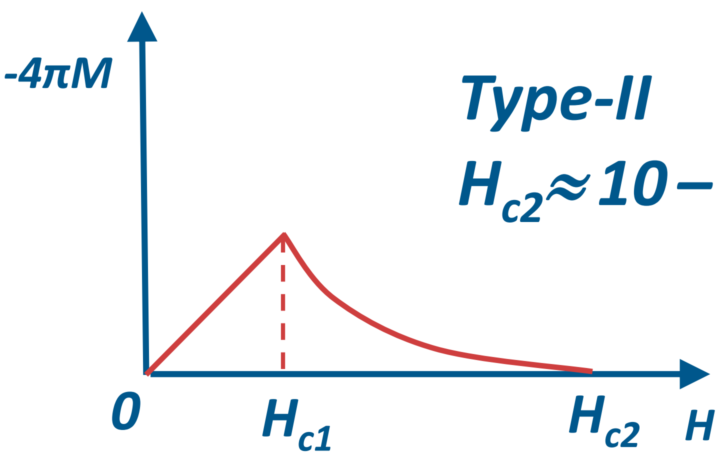


Type-I

$H_c \approx 10^{-3} - 10^{-2}$ T

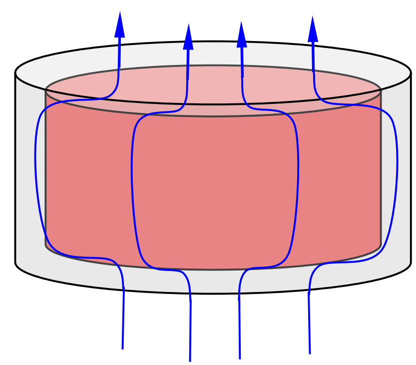


$H < H_c$

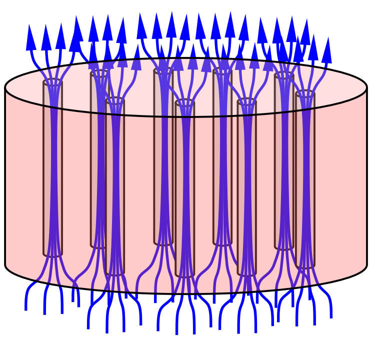


Type-II

$H_{c2} \approx 10 - 10^{2}$ T



$H < H_{c1}$



$H_{c1} < H < H_{c2}$

Previously, in lecture 1

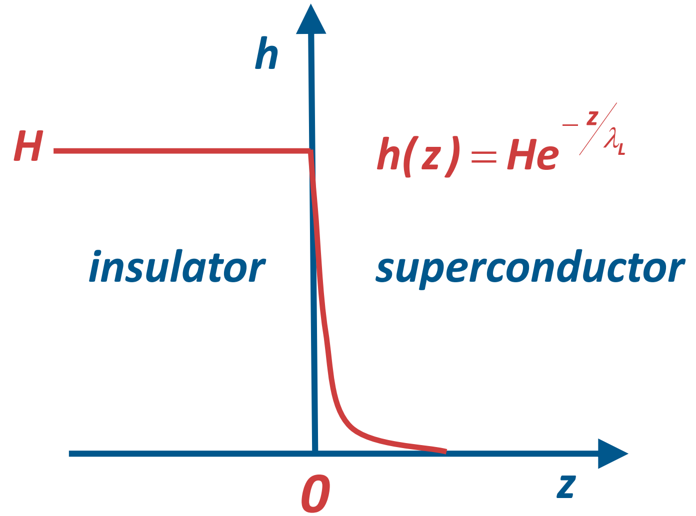
1) $$G_S - G_N = -\frac{H_c^2}{8\pi} \quad \text{if} \quad H < H_c$$

2) $S_S < S_N \Rightarrow$ *higher order in the superconducting state*

3) From the 1[st] London equation

$$\text{with } \lambda_L = \sqrt{\frac{m^* c^2}{4\pi\, n_S e^{*2}}}$$

penetration depth



Landau theory of the order-disorder transition

The order parameter is a quantity which is zero in one phase (usually above the critical point) and non-zero in the other

Example: para- / ferro- magnetic transition

$$\text{order parameter} \quad m(T) = \frac{M(T)}{M(0)} \quad \text{where} \quad M = \frac{1}{V}\sum_i \mu_i$$

$$T > T_{Curie} \Rightarrow m = 0$$

$$T < T_{Curie} \Rightarrow m \neq 0$$

Landau theory of the order-disorder transition

The Landau expansion for the free energy density close to T_{Curie}

$$F(\vec{r},T) = F_0(\vec{r},T) + \alpha\, m^2 + \frac{1}{2}\beta\, m^4$$

At the equilibrium $\dfrac{\partial F}{\partial m} = 0$

$$2\alpha\, m + 2\beta\, m^3 = 0$$

$$m\left(\alpha + \beta\, m^2\right) = 0$$

Two solutions: $m = 0$ **and** $m^2 = -\dfrac{\alpha}{\beta}$

Landau theory of the order-disorder transition

Two solutions: $m = 0$ *and* $m^2 = -\frac{\alpha}{\beta}$

α *and* β *are function of the temperature*

$$\alpha(T) = \alpha_0 + \alpha_1 (T - T_{Curie}) + \frac{1}{2}\alpha_2 (T - T_{Curie})^2 + \ldots$$

$$\beta(T) = \beta_0 + \beta_1 (T - T_{Curie}) + \frac{1}{2}\beta_2 (T - T_{Curie})^2 + \ldots$$

From the minimization conditions

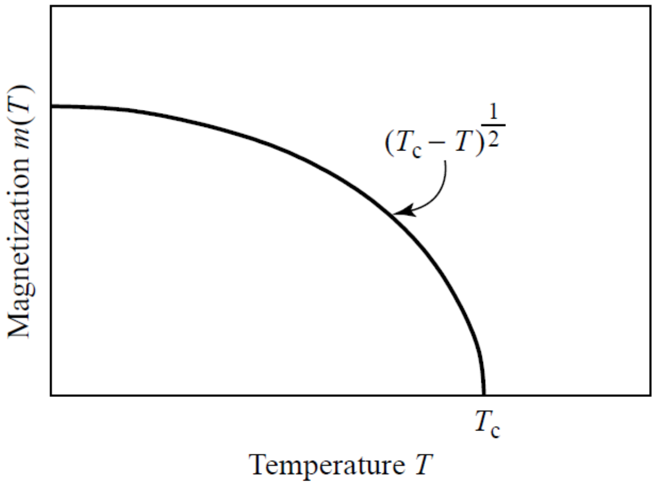
$$\alpha = \alpha_1 \left(T - T_{Curie}\right)$$

$$\beta = \beta_0$$

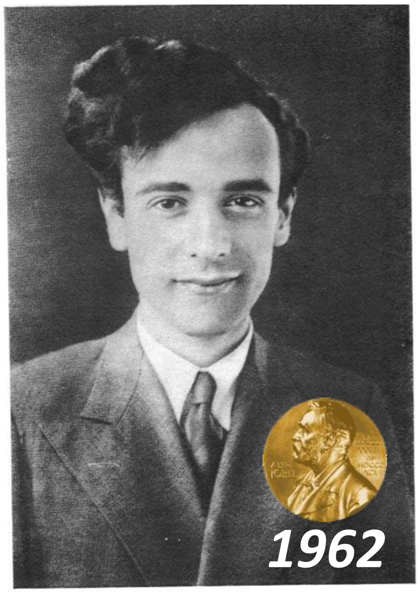
Landau theory of the order-disorder transition

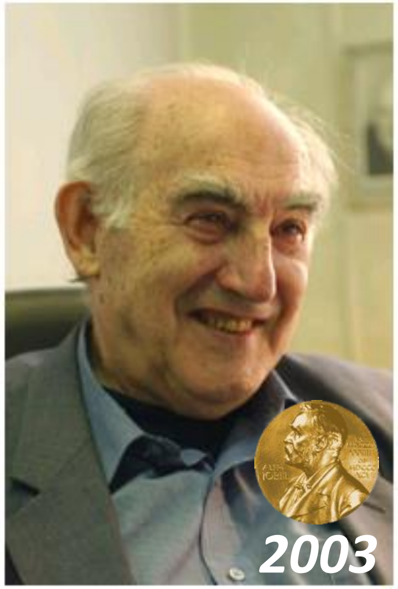
Temperature dependence of the order parameter close to T_{Curie}

$$m^2 = -\frac{\alpha}{\beta} \quad \Rightarrow \quad \boxed{m = \left(\frac{\alpha_1}{\beta_0}\right)^{\frac{1}{2}} \left(T_{Curie} - T\right)^{\frac{1}{2}}}$$



1950: Ginzburg-Landau Theory of Superconductivity





$$F_s = F_n + \text{Condensation energy} + \text{Kinetic energy} + \text{Field energy}$$

Ginzburg-Landau Theory of Superconductivity

The superfluid density n_s is the order parameter

In the London model n_s does not depend on the position

In the G-L theory $n_s(\vec{r}) = |\psi(\vec{r})|^2$ *where* $\psi(\vec{r}) = |\psi(\vec{r})| e^{i\phi}$

Free energy expansion in the zero-field case

$$F_S(\vec{r},T) = F_N(\vec{r},T) + \alpha\, |\psi|^2 + \frac{\beta}{2}\, |\psi|^4 + \frac{\hbar^2}{2m^*} \left|\vec{\nabla}\psi\right|^2$$

rigidity of the order parameter

Ginzburg-Landau Theory of Superconductivity

Free energy expansion in the zero-field case

$$F_S(\vec{r},T) = F_N(\vec{r},T) + \alpha\,|\psi|^2 + \frac{\beta}{2}\,|\psi|^4 + \frac{\hbar^2}{2m^*}\left|\vec{\nabla}\psi\right|^2$$

Free energy expansion for B ≠ 0

In electrodynamics $\vec{p} \rightarrow \vec{p} + \frac{q}{c}\vec{A}$ ***where*** $\vec{B} = \vec{\nabla}\times\vec{A}$

For the momentum operator $-i\hbar\vec{\nabla} \rightarrow -i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}$

$$F_S(\vec{r},T) = F_N(\vec{r},T) + \alpha\,|\psi|^2 + \frac{\beta}{2}\,|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

Ginzburg-Landau Theory of Superconductivity

$$F_s = F_n + \text{Condensation energy} + \text{Kinetic energy} + \text{Field energy}$$

$$F_S(\vec{r},T) = F_N(\vec{r},T) + \alpha\,|\psi|^2 + \frac{\beta}{2}\,|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

Ginzburg-Landau Theory of Superconductivity

Energy density $$F_S(\vec{r},T) = F_N(\vec{r},T) + \alpha\,|\psi|^2 + \frac{\beta}{2}\,|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

Energy $$F_S(T) = \int_V d\vec{r}\; F_S(\vec{r},T)$$

$$F_S(T) = \int_V d\vec{r}\left[F_N(\vec{r},T) + \alpha\,|\psi|^2 + \frac{\beta}{2}\,|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}\right)\psi\right|^2 + \frac{h^2}{8\pi}\right]$$

Minimization procedure ⇒ variations of ψ, ψ^*, and a

$$\delta\,F_S = F_S\left(\psi + \delta\psi,\; \psi^* + \delta\psi^*,\; a + \delta a\right) - F_S\left(\psi, \psi^*, a\right)$$

$$\delta\,F_S = \int_V d\vec{r}\left[(\qquad)\delta\psi + (\qquad)\delta\psi^* + (\qquad)\delta\vec{a}\right]$$

And set $$\frac{\delta F_S}{\delta\psi} = 0 \qquad \frac{\delta F_S}{\delta\psi^*} = 0 \qquad \frac{\delta F_S}{\delta a} = 0$$

Fosshein & Sudbø, Superconductivity: Physics and Applications, pp. 117-120

Next we carry out the minimization procedure for the free energy by making variations in ψ and ψ^*, i.e. by letting $\psi \to \psi + \delta\psi$, and $\psi^* \to \psi^* + \delta\psi^*$. The equilibrium condition $\delta F_s(T) = 0$ will then give us the Ginzburg–Landau equations, two equations of paramount importance in superconductivity. We can immediately write

$$\begin{aligned}\delta F_s(\boldsymbol{r}, T) &= \alpha(\psi + \delta\psi)(\psi^* + \delta\psi^*) - \alpha\psi\psi^* \\ &\quad + \frac{\beta}{2}\left[(\psi + \delta\psi)^2(\psi^* + \delta\psi^*)^2 - \psi^2\psi^{*2}\right] \\ &\quad + \frac{1}{2m}\left[\boldsymbol{G}(\psi + \delta\psi)\boldsymbol{G}^*(\psi^* + \delta\psi^*) - \boldsymbol{G}\psi\boldsymbol{G}^*\psi^*\right] \qquad (4.96)\end{aligned}$$

with $\boldsymbol{G}$ as defined above. Next we simplify:

$$\begin{aligned}\delta F_s(\boldsymbol{r}, T) &= \alpha(\psi\delta\psi^* + \psi^*\delta\psi) + \beta(\psi\,|\psi|^2\,\delta\psi^* + \psi^*\,|\psi|^2\,\delta\psi) \\ &\quad + \frac{1}{2m}(\boldsymbol{G}\psi\boldsymbol{G}^*\delta\psi^* + \boldsymbol{G}\delta\psi\boldsymbol{G}^*\psi^*) \qquad (4.97)\end{aligned}$$

The last parenthesis is

$$\begin{aligned}\boldsymbol{G}\psi\boldsymbol{G}^*\delta\psi^* + \boldsymbol{G}\delta\psi\boldsymbol{G}^*\psi^* &= (-i\hbar\nabla - 2e\boldsymbol{A})\psi(i\hbar\nabla - 2e\boldsymbol{A})\,\delta\psi^* \\ &\quad + (-i\hbar\nabla - 2e\boldsymbol{A})\,\delta\psi(i\hbar\nabla - 2e\boldsymbol{A})\psi^* \qquad (4.98)\end{aligned}$$

Summing up:

$$\begin{aligned}\delta F_s(T) &= \int_V \mathrm{d}^3\boldsymbol{r}\,\Big\{(\alpha\psi\delta\psi^* + \beta\psi\,|\psi|^2\,\delta\psi^* \\ &\quad + \frac{1}{2m}(-i\hbar\nabla - 2e\boldsymbol{A})\psi(i\hbar\nabla - 2e\boldsymbol{A})\,\delta\psi^* + c.c.)\Big\} \qquad (4.99)\end{aligned}$$

After a partial integration we obtain, using $\nabla\boldsymbol{A} = 0$

$$\delta F_s(T) = \int_V \mathrm{d}^3\boldsymbol{r}\left\{[(\alpha\psi + \beta\,|\psi|^2\,\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\boldsymbol{A})^2\psi)]\delta\psi^* + c.c.\right\} \qquad (4.100)$$

Requiring $\delta F_s(T) = 0$ demands that the functions in front of $\delta\psi^*$ and $\delta\psi$ are zero, which means that we have

$$\underline{\text{GL-I:}} \quad \boxed{\alpha\psi + \beta\,|\psi|^2\,\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\boldsymbol{A})^2\psi = 0} \qquad (4.101)$$

This is the first of the two Ginzburg–Landau equations, hereafter referred to as GL-I. Next, as a preparation for the derivation of the second Ginzburg–Landau equation, GL-II, we introduce Maxwell's equation $\nabla \times \boldsymbol{H} = \boldsymbol{J}$ which we use in the form $\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J}$. We write it out as

$$\mu_0\boldsymbol{J} = \nabla \times \boldsymbol{B} = \nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \times \boldsymbol{A}) - \nabla^2\boldsymbol{A} = -\nabla^2\boldsymbol{A} \qquad (4.102)$$

in the London gauge where $\nabla \times \boldsymbol{A} = 0$. When we later encounter the term $-\frac{1}{\mu_0}\nabla^2\boldsymbol{A}$ we will recognize this as the supercurrent $\boldsymbol{J}$.

The procedure by which GL-II is obtained is to vary $\boldsymbol{A}$ in the free energy $F_s(\boldsymbol{r}, t)$, i.e. we let $\boldsymbol{A} \to \boldsymbol{A} + \delta\boldsymbol{A}$ and find the corresponding variation in $F_s(\boldsymbol{r}, t)$. For this minimization of the free energy density $F_s(T)$ with respect to the vector potential $\boldsymbol{A}$ we need only retain the $\boldsymbol{A}$-dependent parts of $F_s(\boldsymbol{r}, t)$. We write this as $F_s(\boldsymbol{r}, t, \boldsymbol{A})$. Next we take the variation

$$\delta F_s(\boldsymbol{r}, t, \boldsymbol{A}) = F_s(\boldsymbol{r}, t, \boldsymbol{A} + \delta\boldsymbol{A}) - F_s(\boldsymbol{r}, t, \boldsymbol{A}) \qquad (4.103)$$

In writing out the expression here we temporarily introduce the symbol $\boldsymbol{p}$ for $-i\hbar\nabla$ to simplify the mathematics. The proper operator is immediately reintroduced in the next step. We find

$$\begin{aligned}\delta F_s(\boldsymbol{r}, t, \boldsymbol{A}) &= \frac{1}{2m}\left[(\boldsymbol{p} - 2e(\boldsymbol{A} + \delta\boldsymbol{A}))\psi\right]\left[(\boldsymbol{p}^* - 2e(\boldsymbol{A} + \delta\boldsymbol{A}))\psi^*\right] \\ &\quad - \frac{1}{2m}\left[(\boldsymbol{p} - 2e\boldsymbol{A})\psi\right]\left[(\boldsymbol{p}^* - 2e\boldsymbol{A})\psi^*\right] \\ &\quad + \frac{1}{2\mu_0}\left[(\nabla \times (\boldsymbol{A} + \delta\boldsymbol{A}))^2 - (\nabla \times \boldsymbol{A})^2\right] \\ &= -\frac{e}{m}\left[i\hbar\psi^*\nabla\psi + i\hbar\psi\nabla\psi^* + 4e\,|\psi|^2\,\boldsymbol{A}\right]\delta\boldsymbol{A} \\ &\quad + \frac{1}{\mu_0}(\nabla \times \delta\boldsymbol{A})(\nabla \times \boldsymbol{A}) \qquad (4.104)\end{aligned}$$

Now $\delta F_s(\boldsymbol{r}, t, \boldsymbol{A})$ is to be integrated over the superconductor volume. We do the last term first:

$$\frac{1}{\mu_0}\int \mathrm{d}^3\boldsymbol{r}(\nabla \times \delta\boldsymbol{A})(\nabla \times \boldsymbol{A}) = -\frac{1}{\mu_0}\int \mathrm{d}^3\boldsymbol{r}\nabla^2\boldsymbol{A} \times \delta\boldsymbol{A} \qquad (4.105)$$

Therefore, the entire $\delta F_s(T)$ becomes

$$\delta F_s(T) = \int \mathrm{d}^3\boldsymbol{r}\left[\frac{ie\hbar}{m}(\psi\nabla\psi^* - \psi^*\nabla\psi) + \frac{4e^2}{m}\,|\psi|^2\,\boldsymbol{A} - \frac{1}{\mu_0}\nabla^2\boldsymbol{A}\right]\delta\boldsymbol{A} = 0 \qquad (4.106)$$

This requires the square bracketed term to be zero. Using the result from Eq. 4.102 the equation for the supercurrent is obtained:

$$\boldsymbol{J} = -\frac{1}{\mu_0}\nabla^2\boldsymbol{A} = \frac{e}{m}\left[(i\hbar\psi^*\nabla\psi - i\hbar\psi\nabla\psi^*) + 4e\,|\psi|^2\,\boldsymbol{A}\right] \qquad (4.107)$$

or

$$\underline{\text{GL-II:}} \quad \boxed{\boldsymbol{J} = \frac{e}{m}\left[\psi^*(-i\hbar\nabla - 2e\boldsymbol{A})\psi + c.c.\right]} \qquad (4.108)$$

This is the second Ginzburg–Landau equation.

The two Ginzburg-Landau equations

$$\frac{\delta F_S}{\delta \psi} = 0 \qquad \frac{\delta F_S}{\delta \psi^*} = 0 \quad \Rightarrow$$

$$\alpha\ \psi + \beta\ |\psi|^2 \psi + \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} - \frac{e^*}{c}\vec{a}\right)^2 \psi = 0$$

1st G-L equation

$$\frac{\delta F_S}{\delta a} = 0 \quad \Rightarrow$$

$$\vec{J} = \frac{e^*\hbar}{2m^* i}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{e^{*2}}{m^* c}|\psi|^2\,\vec{a}$$

2nd G-L equation

Zero-field case deep inside superconductor

The energy density $F_S(\vec{r},T)=F_N(\vec{r},T)+\alpha\ |\psi|^2+\dfrac{\beta}{2}\ |\psi|^4+\dfrac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla}-\dfrac{e^*}{c}\vec{a}\right)\psi\right|^2+\dfrac{h^2}{8\pi}$

becomes $F_S=F_N+\alpha\ |\psi|^2+\dfrac{\beta}{2}\ |\psi|^4$

And the 1st G-L equation $\alpha\ \psi+\beta\ |\psi|^2\psi+\dfrac{1}{2m^*}\left(-i\hbar\vec{\nabla}-\dfrac{e^*}{c}\vec{a}\right)^2\psi=0$

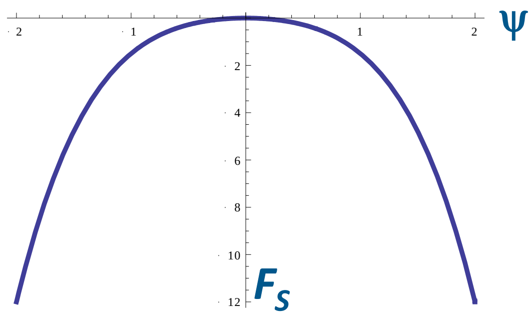
becomes $\alpha\ \psi+\beta\ |\psi|^2\psi=0$

Two solutions: $\psi=0$ and $|\psi|^2=-\dfrac{\alpha}{\beta}$

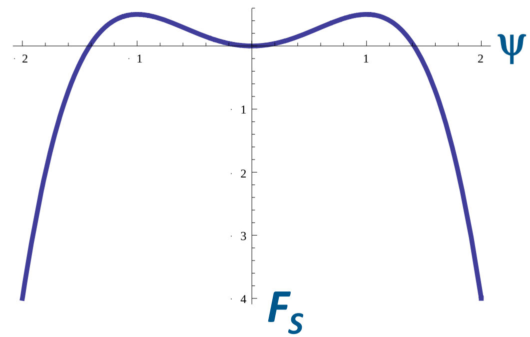
Zero-field case deep inside superconductor

On the sign of α and β

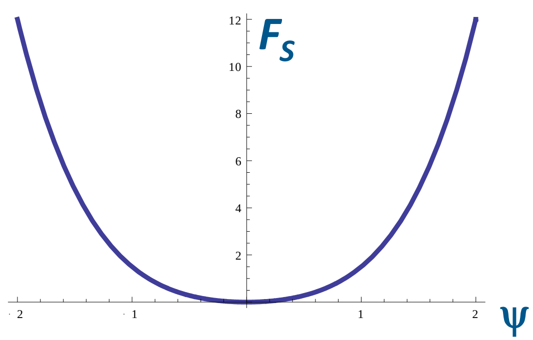
$\alpha \leq 0$ *and* $\beta < 0$



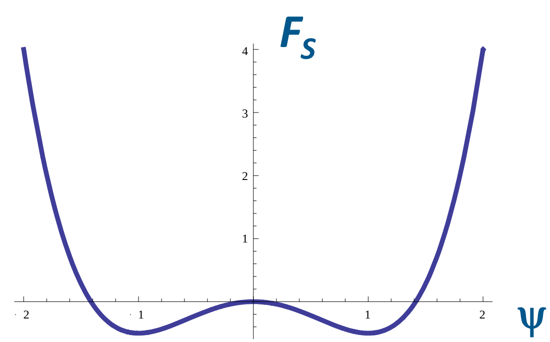
$\alpha > 0$ *and* $\beta < 0$



$\alpha \geq 0$ *and* $\beta > 0$



$\alpha < 0$ *and* $\beta > 0$



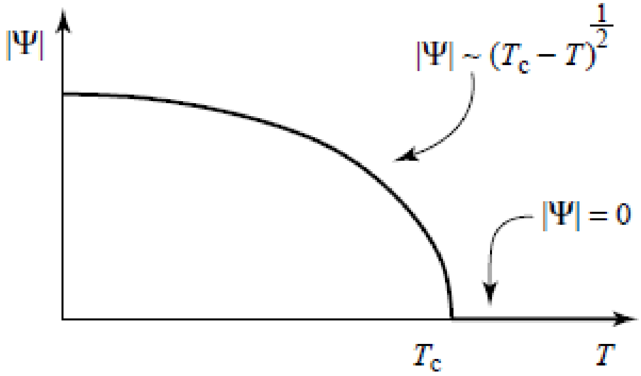
Zero-field case deep inside superconductor

The solution for the superconducting state

$$|\psi|^2 = -\frac{\alpha}{\beta} \qquad \textit{and} \quad F_S - F_N = \alpha\, |\psi|^2 + \frac{\beta}{2}\, |\psi|^4 = -\frac{1}{2}\frac{\alpha^2}{\beta}$$

Choosing $\alpha = \alpha_1 \left(T - T_c\right)$ ***and*** $\beta = \beta_0$

$$|\psi| = \left(\frac{\alpha_1}{\beta_0}\right)^{\frac{1}{2}} \left(T_c - T\right)^{\frac{1}{2}}$$



Zero-field case deep inside superconductor

Relation between α, β and H_c

$$F_S - F_N = \alpha \left|\psi\right|^2 + \frac{\beta}{2}\left|\psi\right|^4 = -\frac{1}{2}\frac{\alpha^2}{\beta}$$

and $$F_S - F_N = -\frac{H_c^2}{8\pi}$$

Thus $$\frac{\alpha^2}{2\beta} = \frac{H_c^2}{8\pi}$$

Zero-field case near superconductor boundary

The 1st G-L equation $$\alpha\,\psi+\beta\,|\psi|^2\psi+\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}-\frac{e^*}{c}\vec{a}\right)^2\psi=0$$

becomes $$\alpha\,\psi+\beta\,|\psi|^2\psi+\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}\right)^2\psi=0$$

We define $f=-\frac{\psi}{\psi_\infty}$ *where* $\psi_\infty^2=-\frac{\alpha}{\beta}$

In one dimension $$\frac{\hbar^2}{2m^*|\alpha|}\frac{d^2f}{dx^2}+f-f^3=0$$

with the boundary condition $f\left(x=0\right)=0$

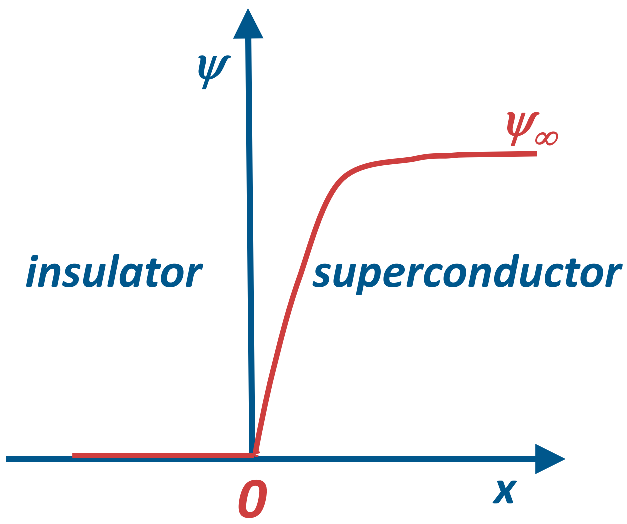
$$\xi^2\frac{d^2f}{dx^2}+f(1-f^2)=0 \quad \textit{with} \quad \xi^2=\frac{\hbar^2}{2m^*|\alpha|}$$

Zero-field case near superconductor boundary

$$\xi^2 \frac{d^2 f}{dx^2} + f(1 - f^2) = 0 \quad \textit{with} \quad \xi^2(T) = \frac{\hbar^2}{2m^* |\alpha(T)|} \propto \frac{1}{1 - T/T_c}$$

The solution is $f = \tanh \frac{x}{\sqrt{2}\xi}$

$$\psi = \psi_\infty \tanh \frac{x}{\sqrt{2}\xi}$$



The coherence length is $\xi = \frac{\hbar}{\sqrt{2m^* |\alpha|}}$

$H \neq 0$ case near superconductor boundary

The 2nd G-L equation $\vec{j} = \frac{e^* \hbar}{2m^* i}\left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*\right) - \frac{e^{*2}}{m^* c}|\psi|^2 \vec{a}$

If ξ is small $\vec{j} \approx -\frac{e^{*2}}{m^* c}|\psi|^2 \vec{a}$

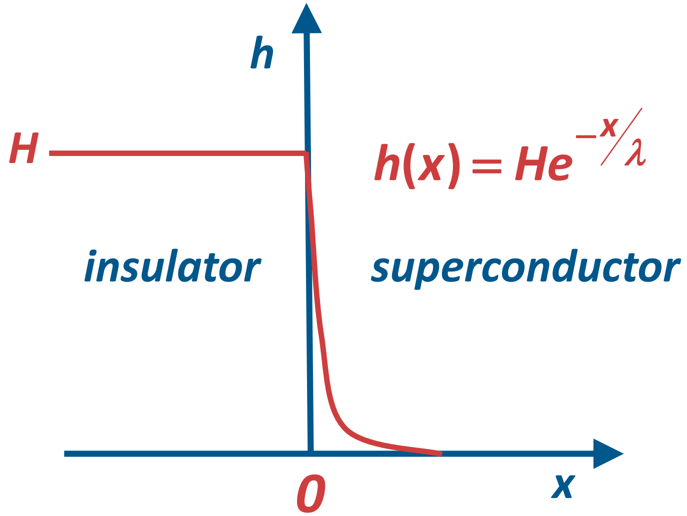
see C.P. Poole, Superconductivity (2007), pp. 150-154

$$\vec{\nabla} \times \vec{j} = -\frac{e^{*2}}{m^* c}|\psi|^2 \vec{\nabla} \times \vec{a} = -\frac{e^{*2}}{m^* c}|\psi|^2 \vec{h}$$

$\vec{\nabla} \times \vec{h} = \frac{4\pi}{c}\vec{j} \Rightarrow$ $\vec{h} - \lambda^2 \nabla^2 \vec{h} = 0$ with $\lambda = \sqrt{\frac{m^* c^2}{4\pi|\psi|^2 e^{*2}}}$

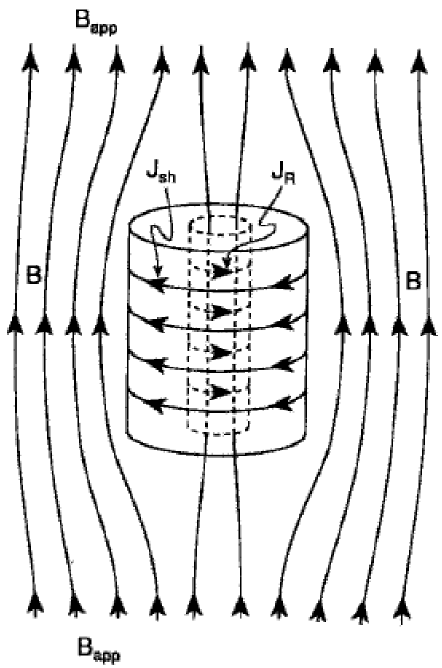
$H \neq 0$ case near superconductor boundary

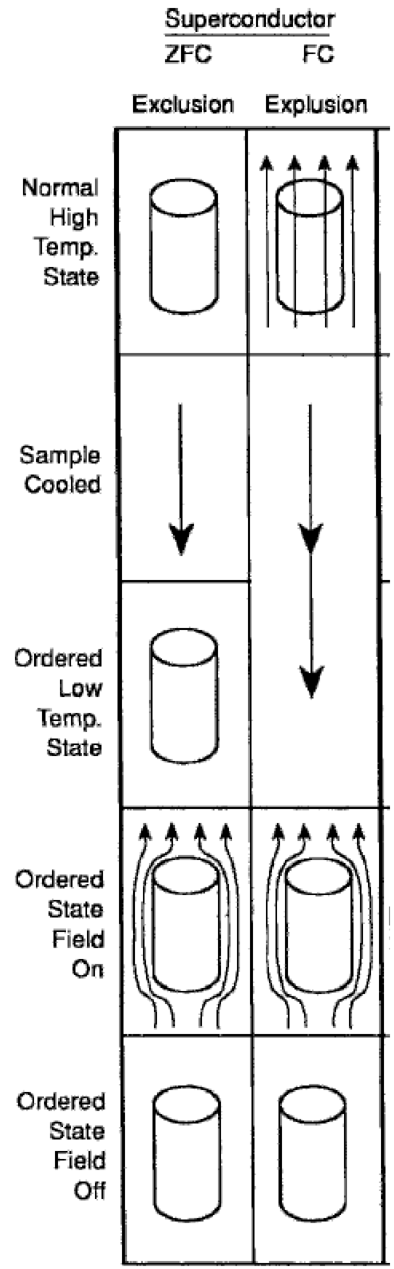
$$\vec{h} - \lambda^2 \nabla^2 \vec{h} = 0 \quad \textit{with} \quad \lambda = \sqrt{\frac{m^* c^2}{4\pi |\psi|^2 e^{*2}}}$$



The London penetration depth is $\lambda_L = \sqrt{\frac{m^* c^2}{4\pi n_S e^{*2}}}$

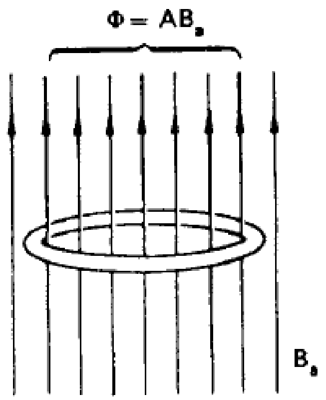
Flux exclusion and retention





Flux exclusion & retention : the resistenceless circuit

The total magnetic flux threading a closed resistanceless circuit cannot change so long as the circuit remains resistanceless.



(a)

1. ***The circuit is cooled below T_c in an applied field B_a. The magnetic flux in the circuit is $\Phi = A\,B_a$***

2. ***The value of B_a is changed***

$$-A\frac{dB_a}{dt} = \cancel{RI} + L\frac{dI}{dt} \quad \Rightarrow LI(t) + AB_a(t) = \text{constant}$$

Flux quantization

The 2nd G-L equation $\vec{J} = \frac{e^{*}\hbar}{2m^{*}i}\left(\psi^{*}\vec{\nabla}\psi - \psi\vec{\nabla}\psi^{*}\right) - \frac{e^{*2}}{m^{*}c}|\psi|^{2}\vec{a}$

$$\psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})} \quad \Rightarrow \quad \psi^{*}\vec{\nabla}\psi - \psi\vec{\nabla}\psi^{*} = 2i|\psi|^{2}\vec{\nabla}\phi(\vec{r})$$

$$\vec{J} = \frac{e^{*}\hbar}{2m^{*}i}\left(2i|\psi|^{2}\vec{\nabla}\phi\right) - \frac{e^{*2}}{m^{*}c}|\psi|^{2}\vec{a}$$

$$\vec{a} + \frac{m^{*}c}{e^{*2}|\psi|^{2}}\vec{J} = \frac{\hbar c}{e^{*}}\vec{\nabla}\phi$$

$$\oint_{\Gamma}\vec{a}d\vec{l} + \frac{m^{*}}{e^{*2}}\oint_{\Gamma}\frac{\vec{J}d\vec{l}}{|\psi|^{2}} = \frac{\hbar c}{e^{*}}\oint_{\Gamma}\vec{\nabla}\phi\cdot d\vec{l}$$

Flux quantization

$$\oint_{\Gamma} \vec{a}\,d\vec{l} + \frac{m^*}{e^{*2}} \oint_{\Gamma} \frac{\vec{J}\,d\vec{l}}{|\psi|^2} = \frac{\hbar c}{e^*} \oint_{\Gamma} \vec{\nabla}\phi \cdot d\vec{l}$$

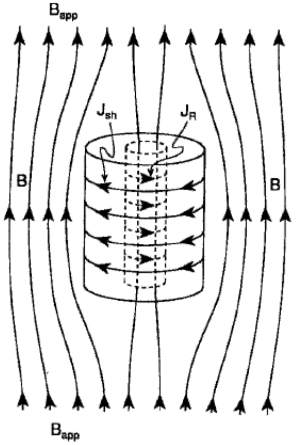
$$\oint_{\Gamma} \vec{a}\,d\vec{l} = \int_S \left(\vec{\nabla} \times \vec{a}\right) \hat{n}\,dS = \int_S \vec{h}\,\hat{n}\,dS = \Phi_S(\vec{h})$$

$$\oint_{\Gamma} \vec{\nabla}\phi \cdot d\vec{l} = 2\pi n$$

ψ is a single-value function

$$\Phi_S(\vec{h}) + \frac{m^*}{e^{*2}} \oint_{\Gamma} \frac{\vec{J}\,d\vec{l}}{|\psi|^2} = \frac{hc}{e^*} n$$

$$\Phi_S(\vec{h}) = \frac{hc}{e^*} n = n\Phi_0$$

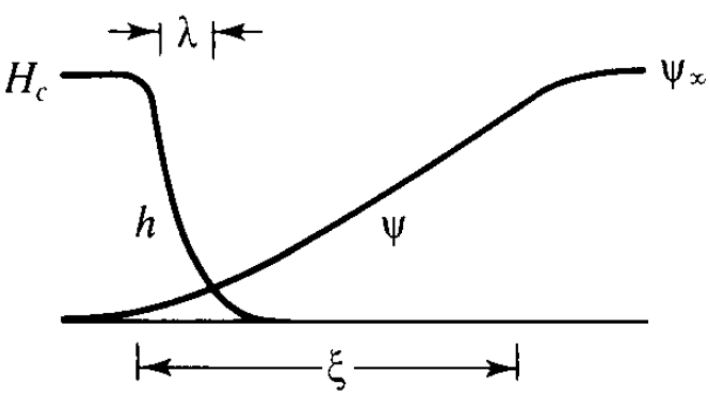


Calculation of the domain-wall energy

At a domain wall

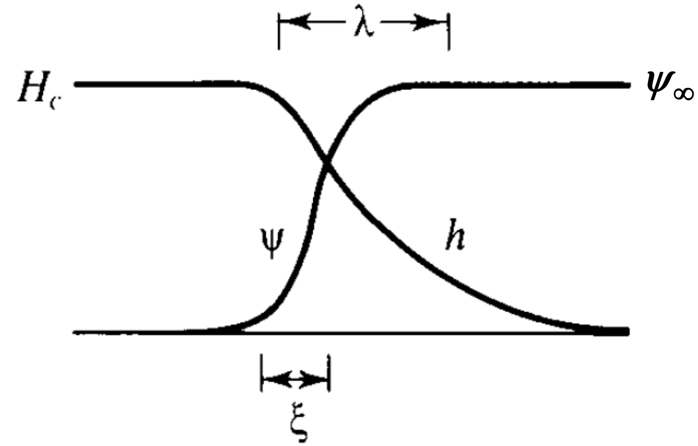
- magnetic energy is gained $E_1 = -\lambda A \frac{H_c^2}{8\pi}$
- condensation energy is lost $E_2 = \xi A \frac{H_c^2}{8\pi}$

$\lambda << \xi$



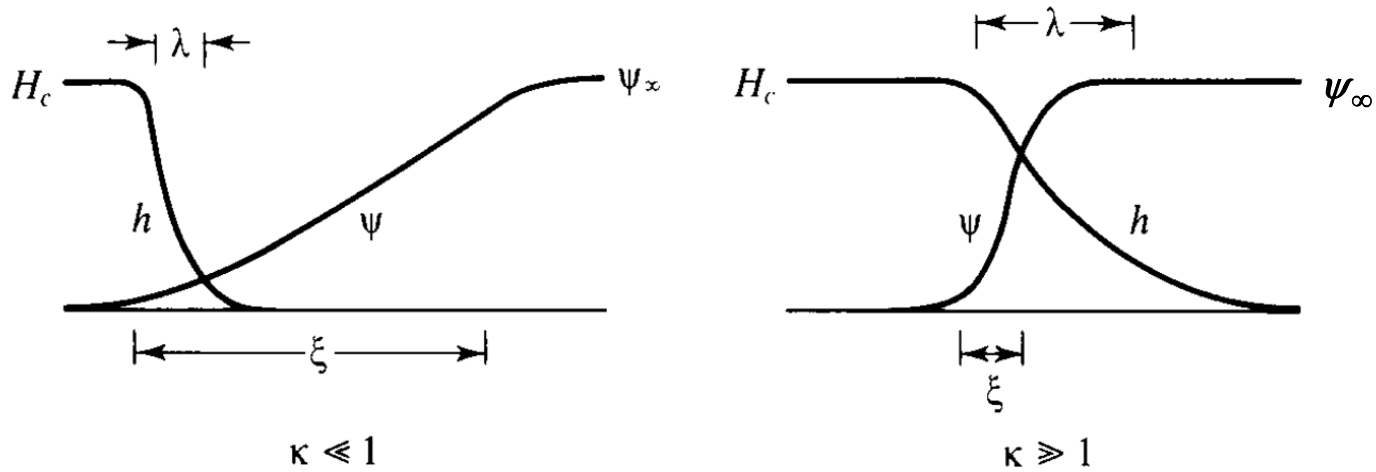
$$\Delta E = A\frac{H_c^2}{8\pi}\left(\xi - \lambda\right) > 0$$

$\xi << \lambda$



$$\Delta E = A\frac{H_c^2}{8\pi}\left(\xi - \lambda\right) < 0$$

Calculation of the domain-wall energy



Ginzburg-Landau parameter $\kappa = \dfrac{\lambda}{\xi}$

$$\kappa < \frac{1}{\sqrt{2}} \Rightarrow \Delta E > 0$$

Type-I superconductor

$$\kappa > \frac{1}{\sqrt{2}} \Rightarrow \Delta E < 0$$

Type-II superconductor

Bibliography

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