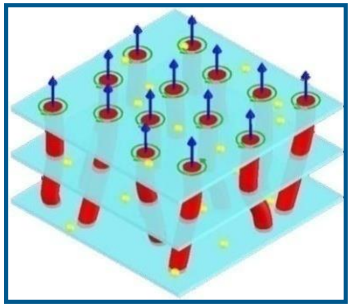
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Superconductivity and its applications

Lecture 1



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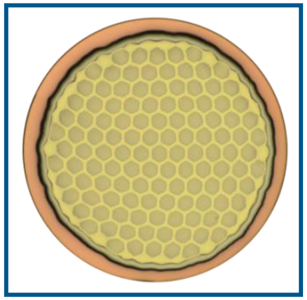
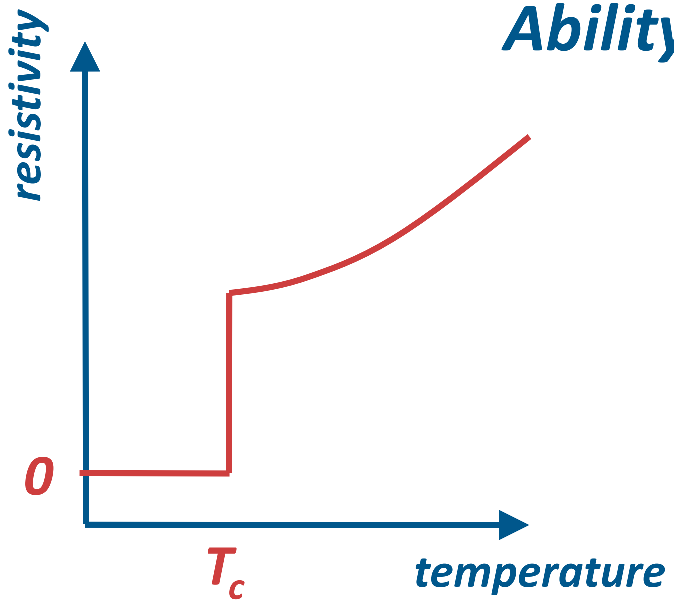


Scope & Summary

- *Phenomenology*
- *An introduction to electrodynamics of superconductors*
- *The Ginzburg-Landau theory*
- *Interactions in the vortex line system, focused on pinning phenomena, critical state and thermal effects on vortex dynamics*
- *An overview of the superconducting materials, addressing also the properties of superconducting wires and cables*
- *Superconductor Technology: basic design and operation issues of a superconducting device*

Discovery of Superconductivity in 1911



Ability to carry a current without dissipation

- Transport of energy
- Generation of high magnetic fields

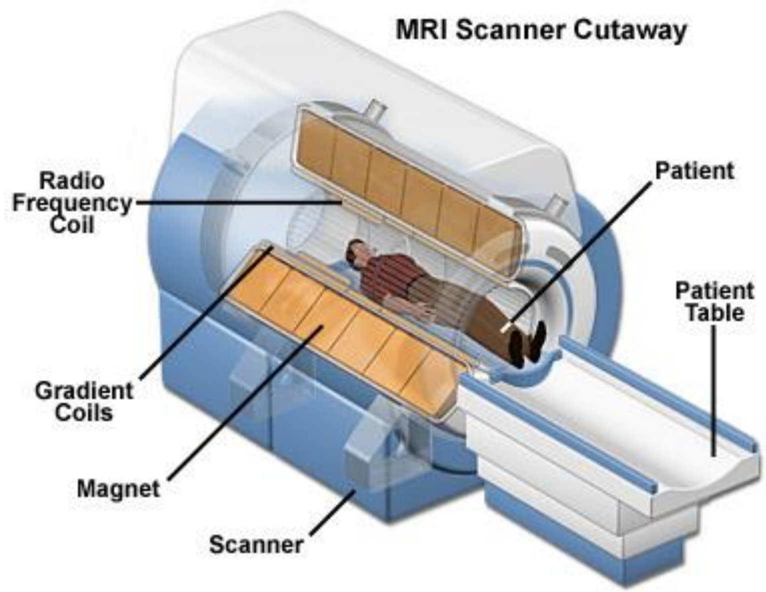
Drawbacks:

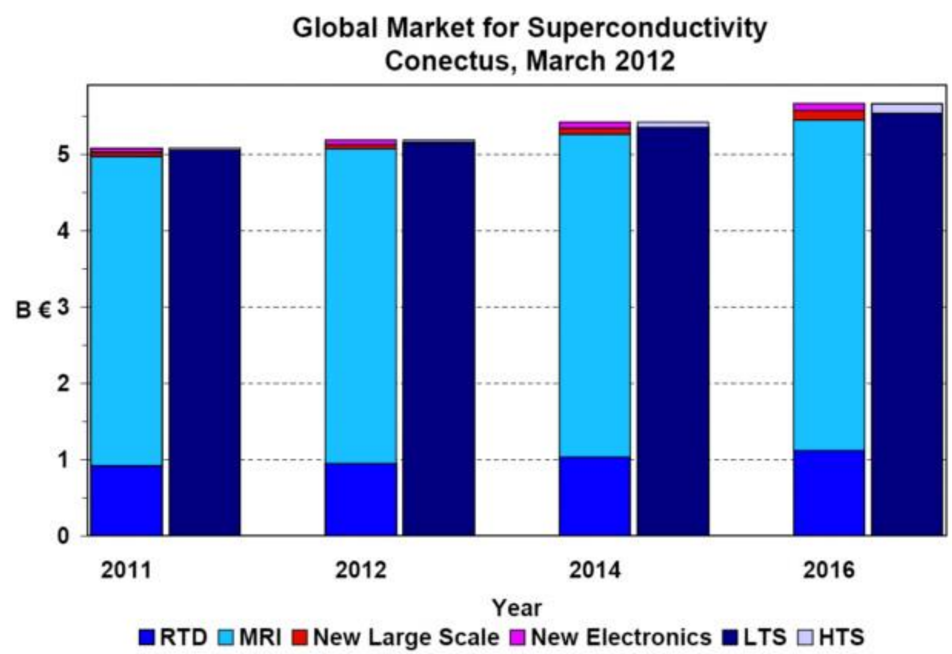
Loss-less currents cannot exceed the critical current I_c

1000+ superconducting compounds, very few for practical use

Superconductors Today

Medical Imaging and NMR spectroscopy

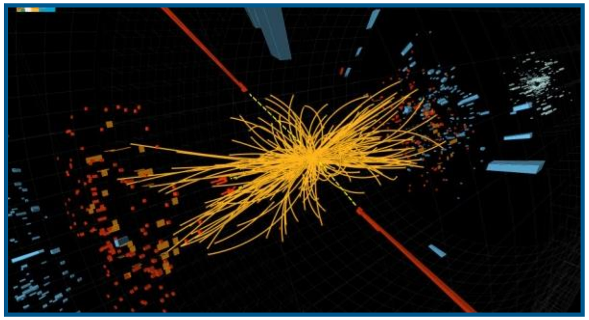




Superconductors Today

Medical Imaging and

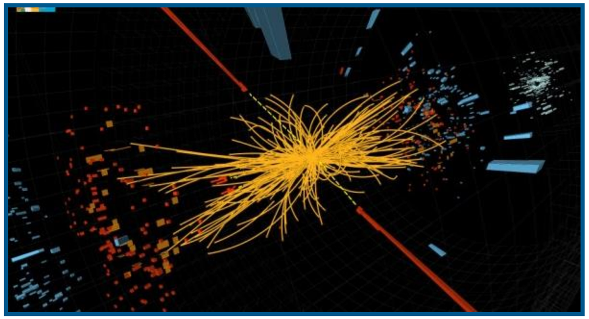
NMR spectroscopy



Superconductivity and LHC

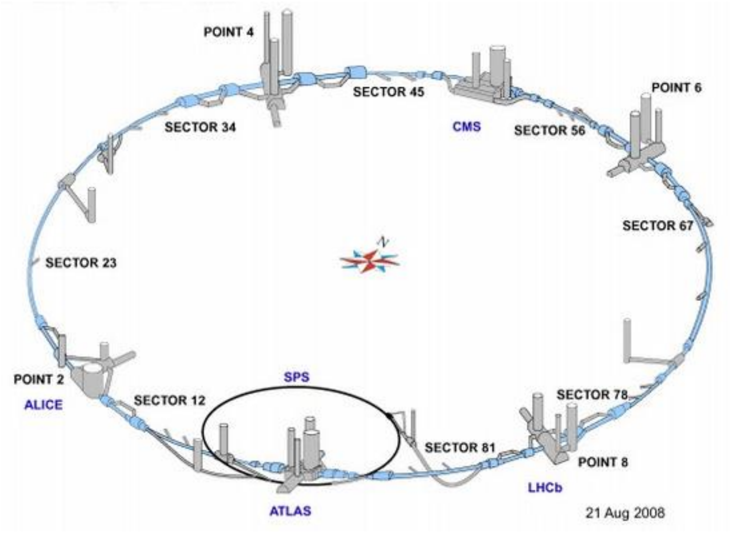
Higgs boson

Superconductors Today



Superconductivity and LHC

Higgs boson



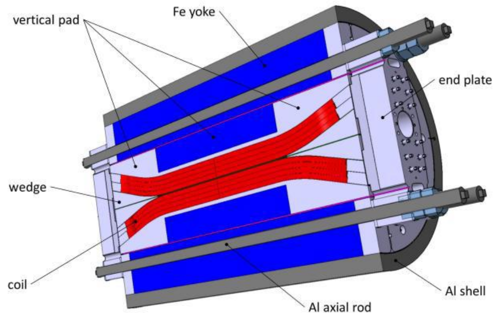
Along the 27 Km of LHC

1200 dipole magnets

400 quadrupole magnets

1200+ tonnes of NbTi @ 1.9K

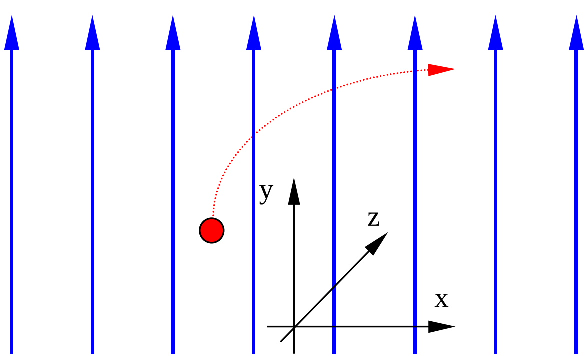
Superconductors Today



Superconductivity and LHC

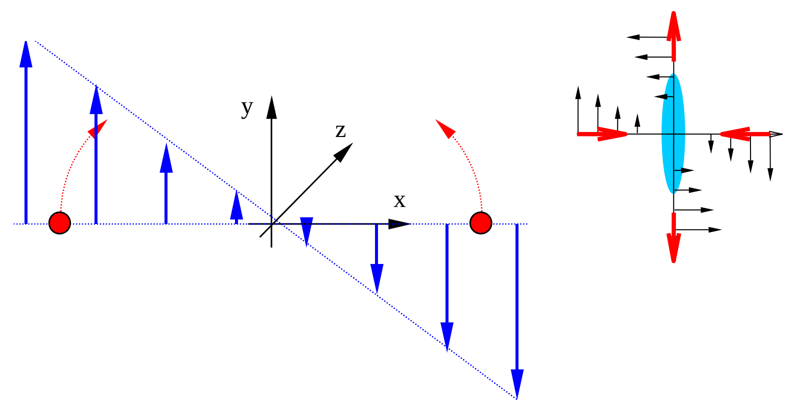
Bending the beam

Uniform field (dipole)

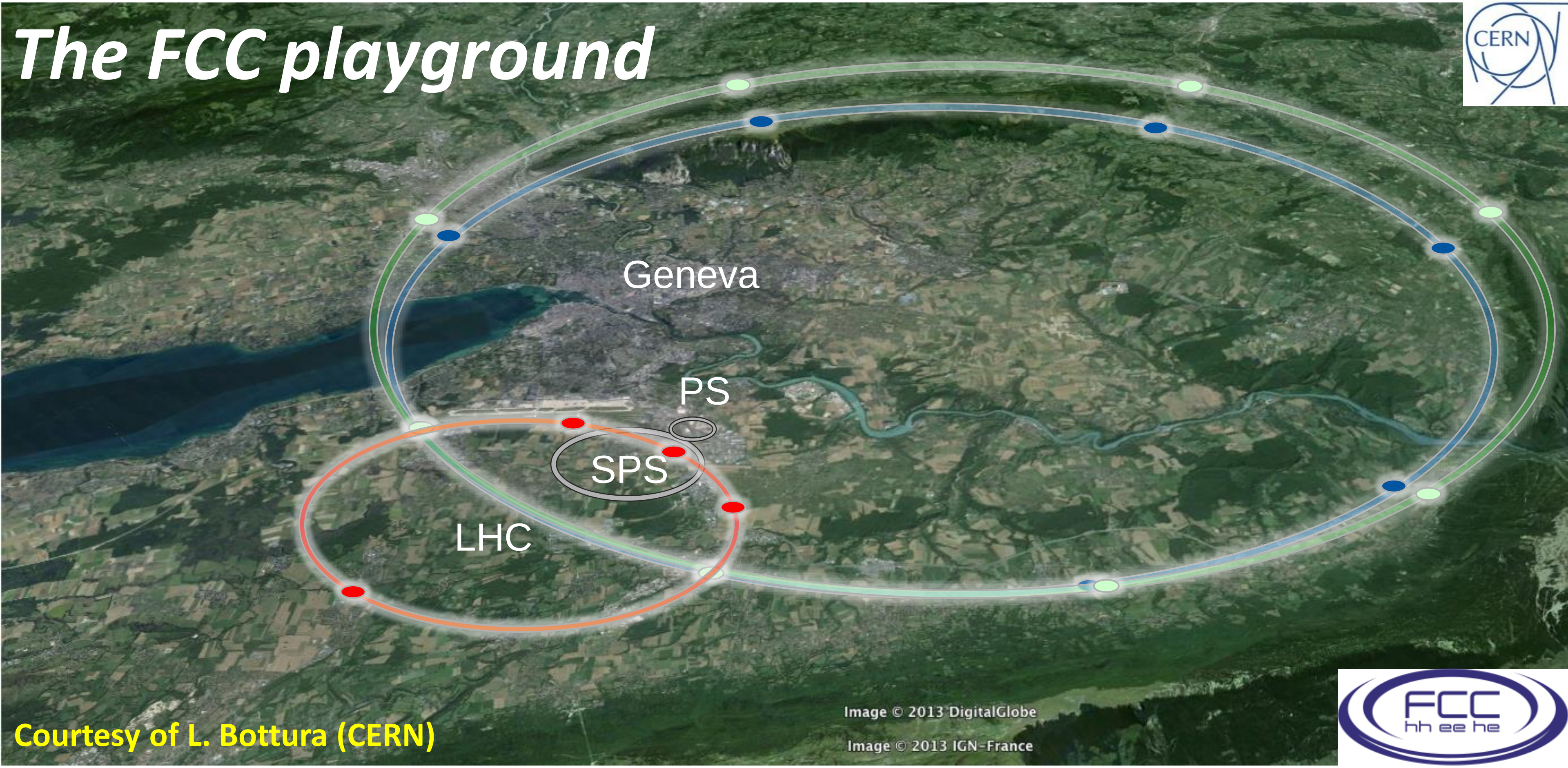


Focusing the beam

Gradient field (quadrupole)



The idea of a Future Circular Collider @ 100 TeV

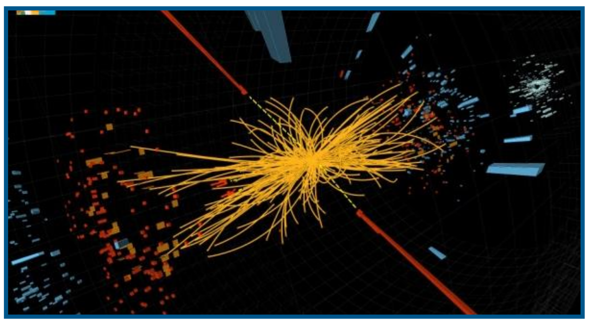


LHC	HE-LHC	FCC-hh	FCC-hh
27 km, 8.33 T	27 km, 20 T	80 km, 20 T	100 km, 16 T
14 TeV (c.o.m.)	33 TeV (c.o.m.)	100 TeV (c.o.m.)	100 TeV (c.o.m.)
1300 tons NbTi	3000 tons LTS	9000 tons LTS	6000 tons Nb_3Sn
	700 tons HTS	2000 tons HTS	3000 tons NbTi

Superconductors Today

Medical Imaging and

NMR spectroscopy



Superconductivity and LHC

Higgs boson

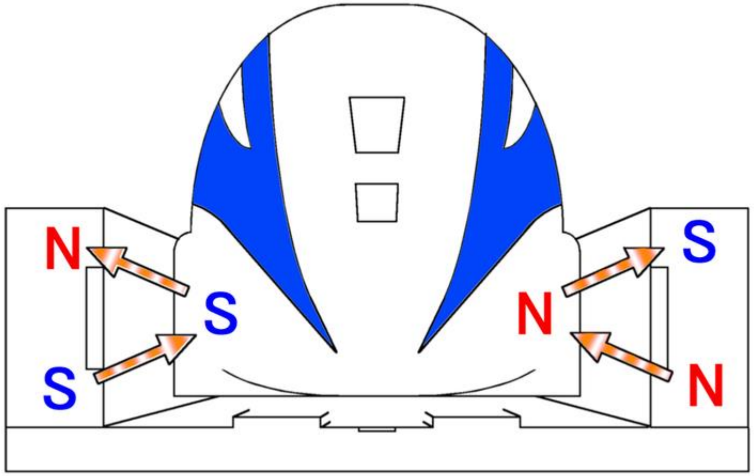
High speed trains

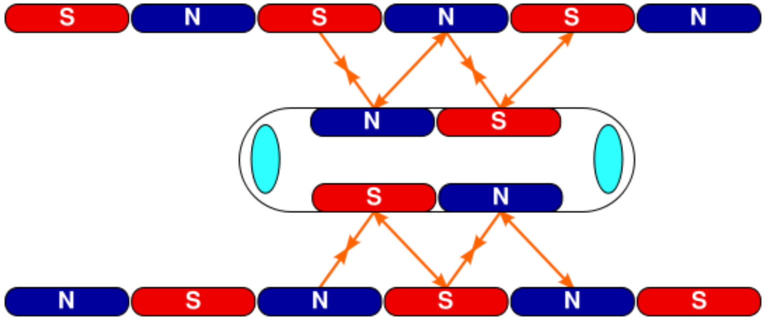
by magnetic levitation

Superconductors Today

High speed trains

by magnetic levitation

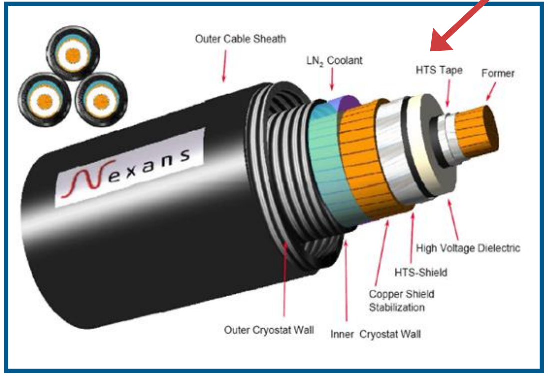




The line between Tokyo and Osaka (550 Km) is under construction

Superconductors Tomorrow

Power distribution, generation and storage



superconducting cables

fault current limiters

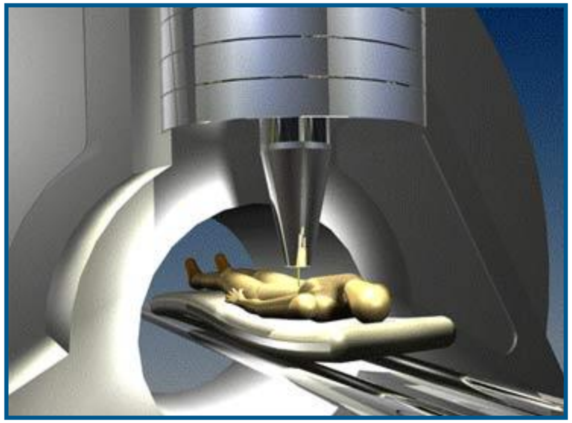
20% from renewable sources by 2020

$$E = \frac{1}{2}\frac{B^2}{\mu}$$

energy density of the magnetic field

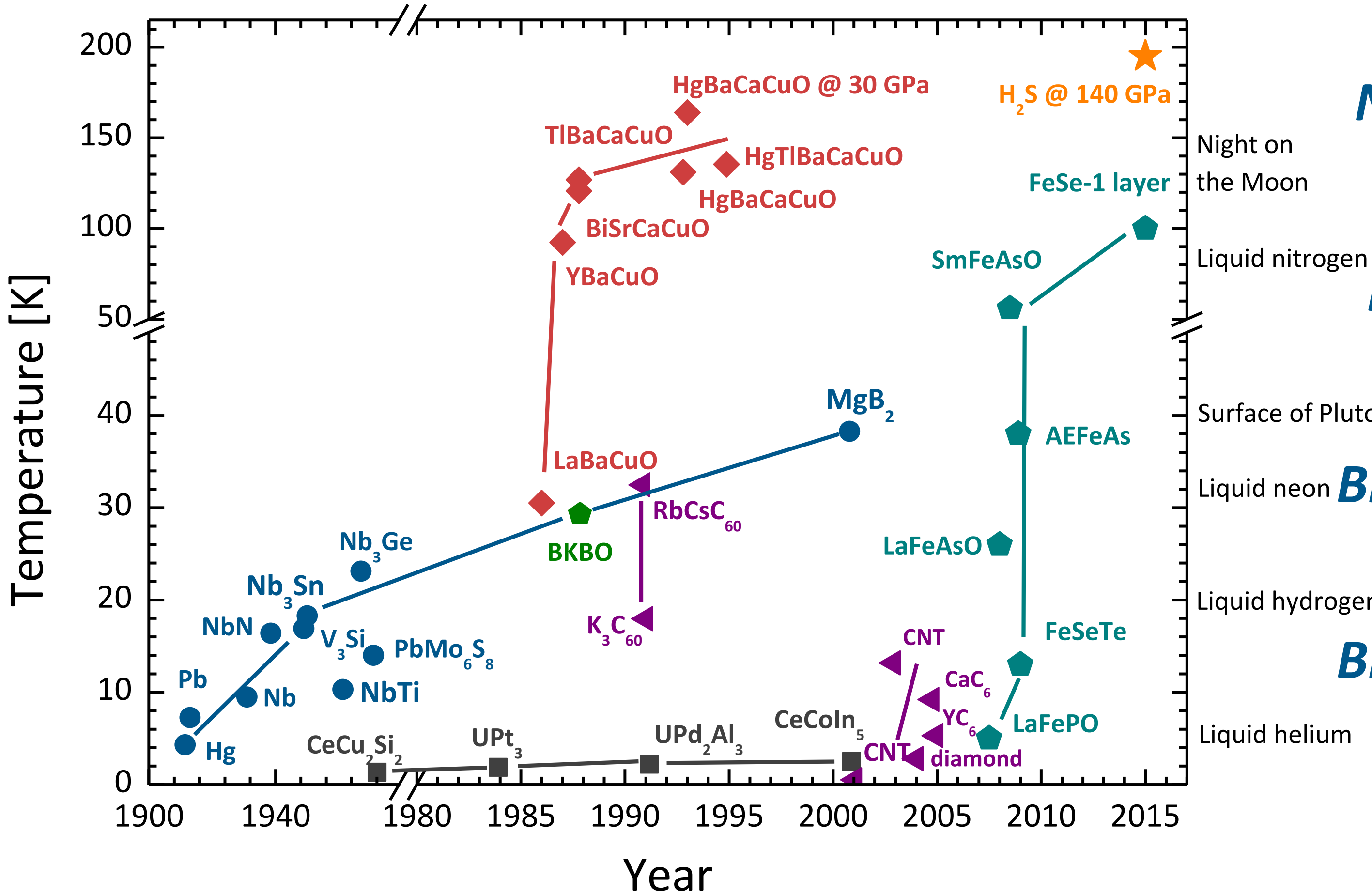
New medical applications:

Compact accelerators for hadron therapy

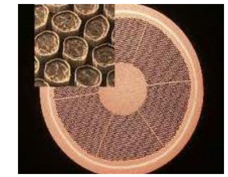


A big step to get there…

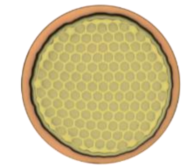
Superconductors History



NbTi

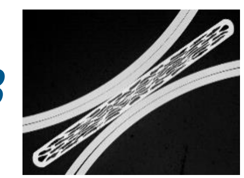


Nb_3Sn

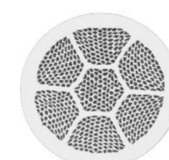


MgB_2

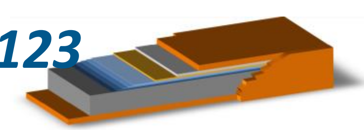
Bi2223



Bi2212



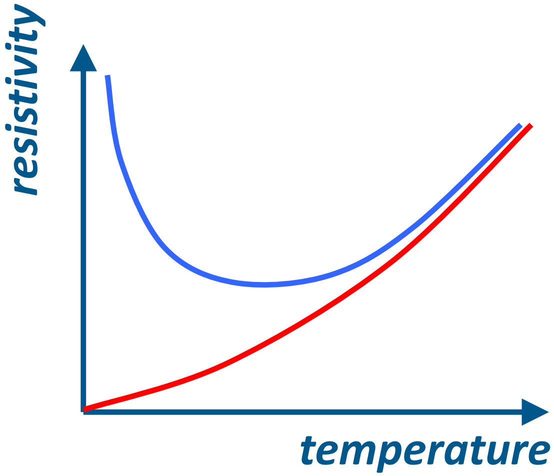
Y123



Superconductors Pre-History

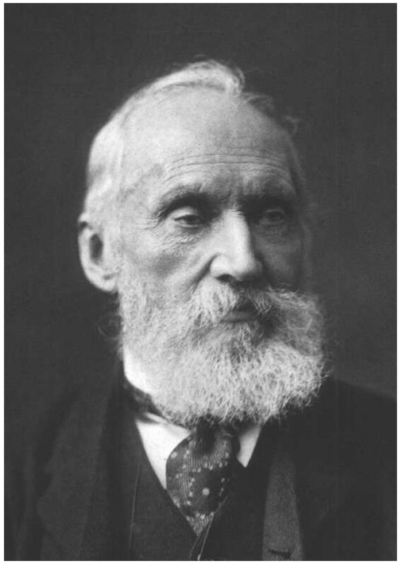
A great physics problem in 1900:

What is the limit of electrical resistivity at the absolute zero ?



… electrons flowing through a conductor would come to a complete halt or, in other words, metal resistivity will become infinity at absolute zero.

W. Thomson (Lord Kelvin)

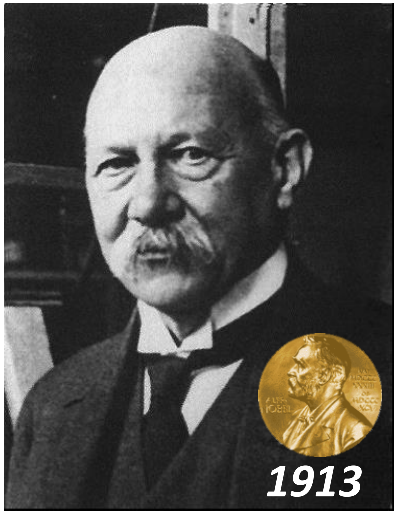


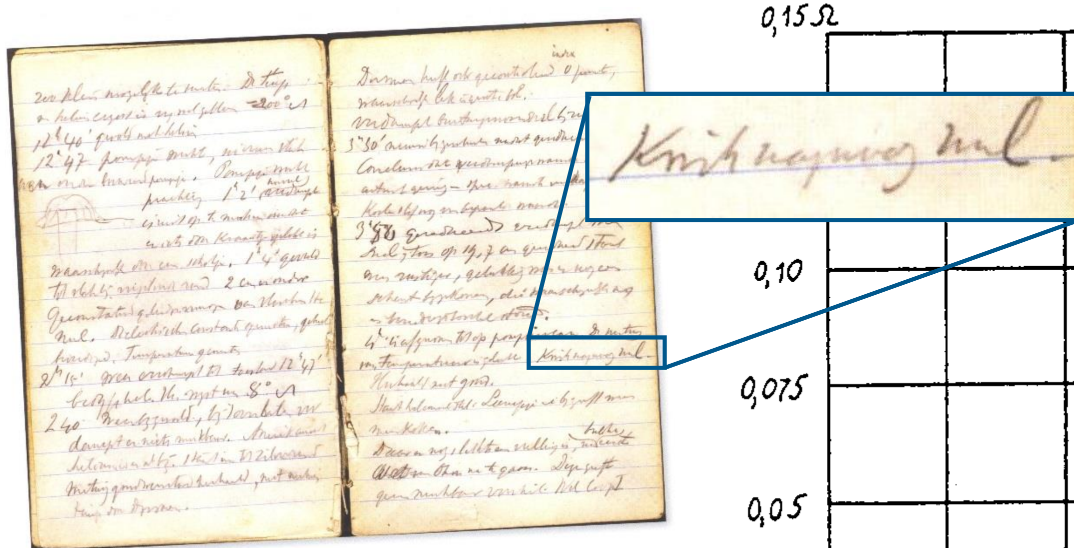
"X-rays are an hoax"

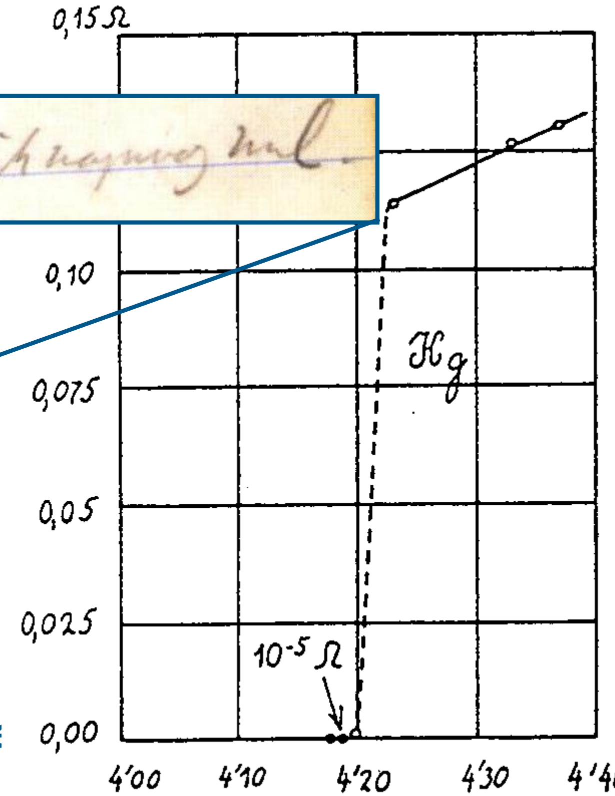
"I have not the smallest molecule of faith in aerial navigation other than ballooning or of expectation of good results from any of the trials we hear of"

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement"

Superconductors Early History







… thus the mercury at 4.2 K has entered a new state, which, owing to its particular electrical properties, can be called the state of superconductivity…

H. Kamerlingh-Onnes (1911)

Superconductors Early History

Onnes in 1913 conceived a 10 T magnet

What he pointed out:

- ***The impossibility of doing this with Cu cooled by liquid air (as expensive as a warship)***
- ***The possibility of doing it with superconductor (1000 A/mm^2 with a Hg wire, 460 A/mm^2 with a Pb wire)***

A « little » problem!

- ***Resistance developed at 0.8 A, not 20 A***
- ***48 years had to go by before the path to high field superconducting magnets was cleared***

The great silence: 1914-1961

International Conference on High Magnetic Fields, Massachusetts Institute of Technology, November 1961

Who	Field	Material	Bore
Bell	6.9 T	Nb_3Sn	0.25″
Atomics International	5.9 T	Nb25Zr	0.5″
Westinghouse	5.6 T	Nb25Zr	0.15″

Abolish Ohm's law !

$\vec{J} = \sigma\,\vec{E}$ **holds also for superconductors**

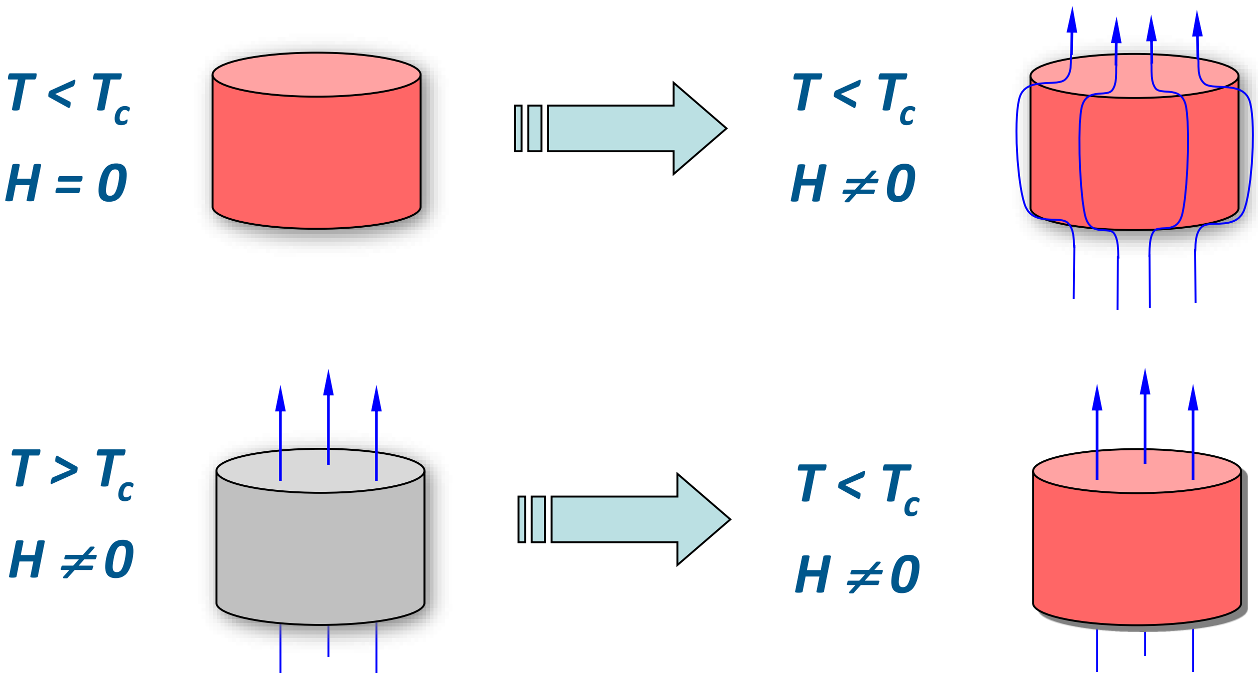
$$\rho = \mathbf{0} \;\Rightarrow\; \sigma = \infty$$

$J \neq \infty$, J is finite $\Rightarrow$ $E = 0$

From the Maxwell equation $\vec{\nabla} \times \vec{E} = -\dfrac{1}{c}\dfrac{\partial \vec{B}}{\partial t}$

$$\boxed{\vec{E} = 0 \;\Rightarrow\; \frac{\partial \vec{B}}{\partial t} = 0 \;\Rightarrow\; \vec{B} = const.}$$

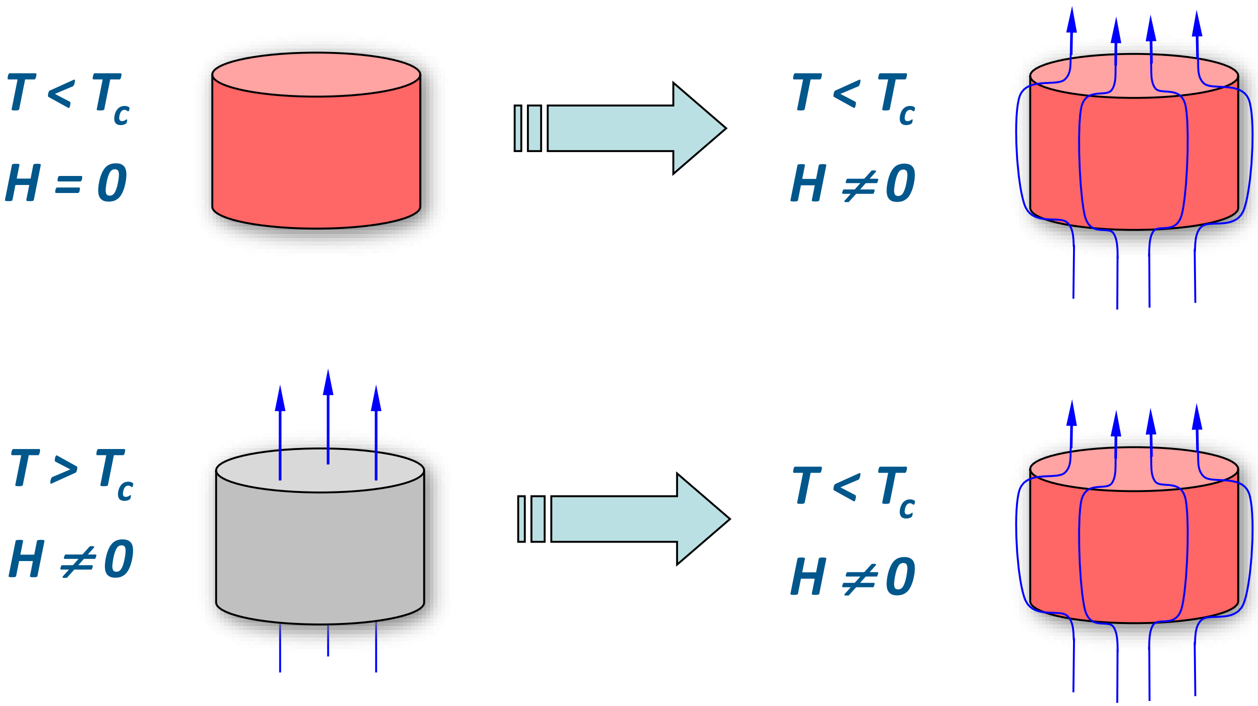
B = constant



This would imply that superconductivity is not a thermodynamic state !!

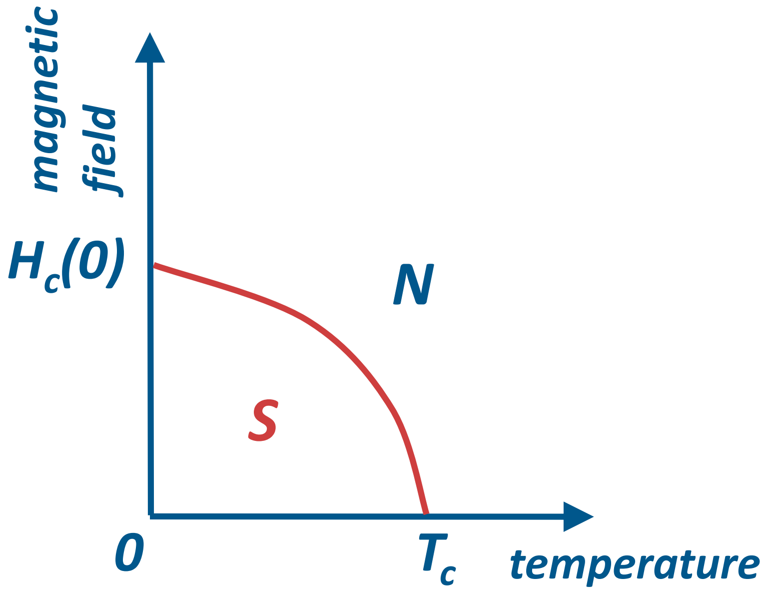
Some ingredients are missing...

The Meissner–Ochsenfeld effect - B = 0 !



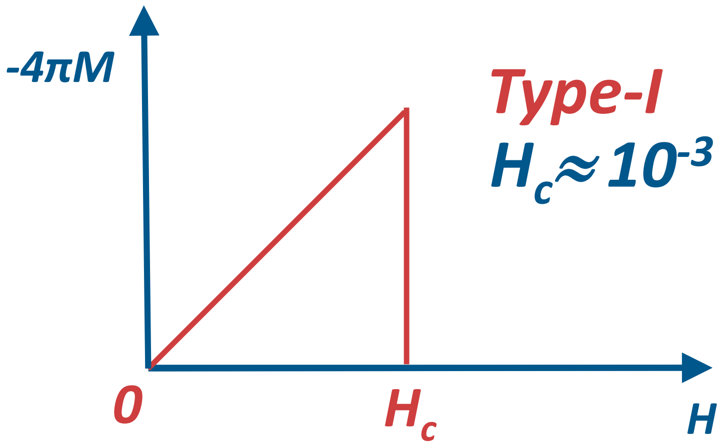
Superconductor = Perfect Conductor + Perfect Diamagnetism

Superconducting Phase Diagram



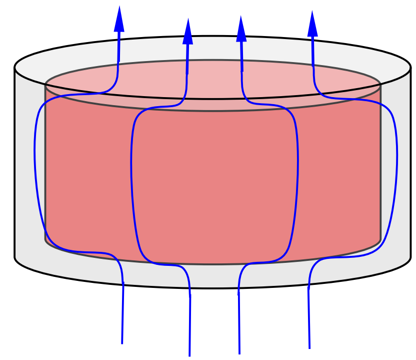
$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Type-I and Type-II superconductors

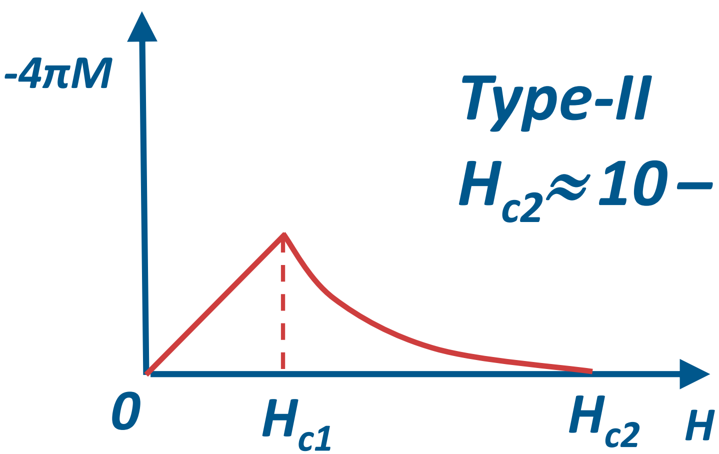


Type-I

$H_c \approx 10^{-3} - 10^{-2}$ T

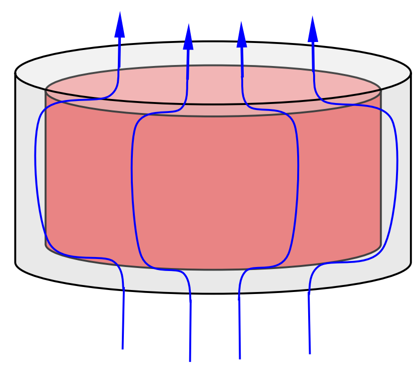


$H < H_c$

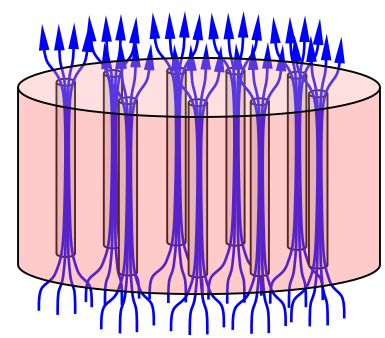


Type-II

$H_{c2} \approx 10 - 10^{2}$ T



$H < H_{c1}$



$H_{c1} < H < H_{c2}$

$-4\pi M = H - B$

Thermodynamics of the superconducting state

$$U(S,V,B) \underset{Legendre\ \ transformation}{\Rightarrow} F(T,V,B) \underset{Legendre\ \ transformation}{\Rightarrow} G(T,P,H)$$

The differential of the Gibbs free energy is

$$dG = VdP - SdT - \frac{1}{4\pi} BdH$$

Thermodynamics of the superconducting state

1st case: $T = T'$, normal state, field sweep from 0 to H^*

$$G_N(T',H^*) - G_N(T',0) = -\frac{1}{4\pi}\int_0^{H^*} BdH = -\frac{H^{*2}}{8\pi}$$

2nd case: $T = T''$, superconducting state, field sweep from 0 to $H^* < H_c$

$$G_S(T'',H^*) - G_S(T'',0) = 0$$

At $H = H_c$ $\quad G_S(T,H_c) = G_N(T,H_c) = G_N(T,0) - \frac{H_c^2}{8\pi}$

and thus $G_S(T,0) = G_N(T,0) - \frac{H_c^2}{8\pi}$

Entropy: ΔS @ H = 0 and T < T_c

$$S = -\left(\frac{\partial G}{\partial T}\right)_{P,H}$$

$$S_N - S_S = -\left[\frac{\partial}{\partial T}\left(G_N - G_S\right)\right] = -\left[\frac{d}{dT}\frac{H_c^2}{8\pi}\right] = -\frac{1}{4\pi}H_c\frac{dH_c}{dT}$$

$$H_c\left(T\right) = H_c\left(0\right)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \Rightarrow \frac{dH_c}{dT} < 0 \Rightarrow S_S < S_N$$

Specific heat: Δc @ T_c

$$c = T\left(\frac{\partial S}{\partial T}\right)_{P,H}$$

$$c_S - c_N = T\left[\frac{\partial}{\partial T}\left(S_S - S_N\right)\right] \quad = T\left[\frac{d}{dT}\left(\frac{1}{4\pi}H_c\frac{dH_c}{dT}\right)\right]$$

$$= \frac{T}{4\pi}\left[\left(\frac{dH_c}{dT}\right)^2 + H_c\frac{d^2H_c}{dT^2}\right]$$

At $T = T_c$ $\quad c_S - c_N\Big|_{T_c} = \frac{T_c}{4\pi}\left(\frac{dH_c}{dT}\right)^2$

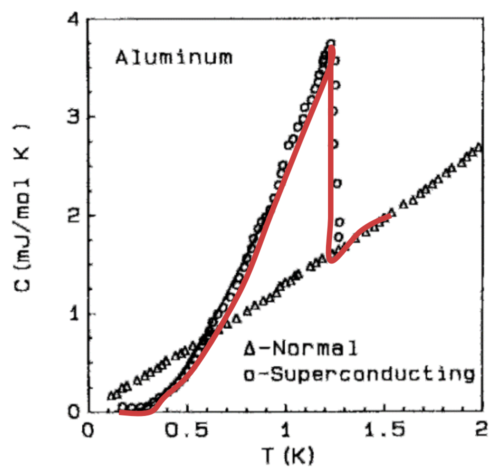
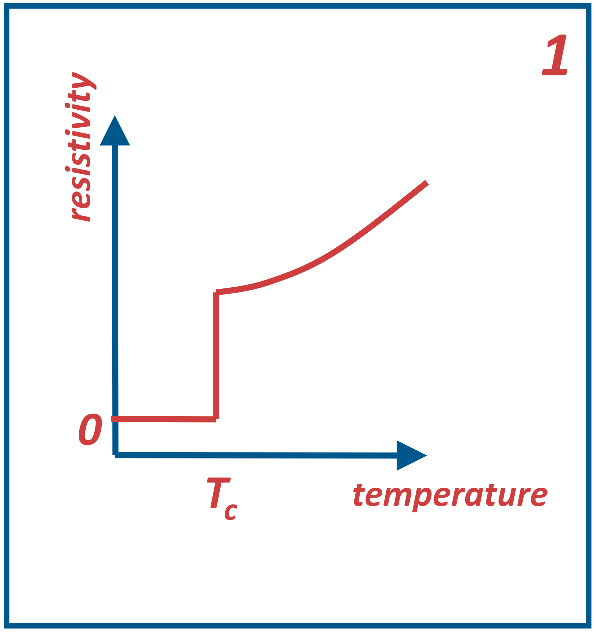
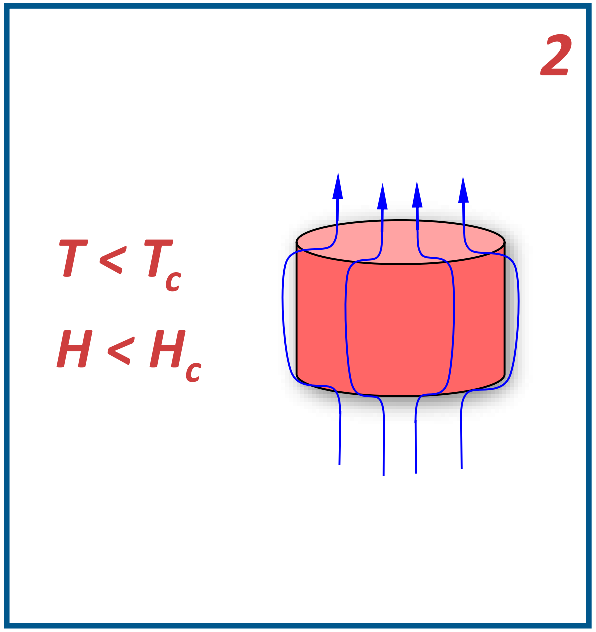


Figure 4.5 Specific heat jump in superconducting Al compared with the normal-state specific heat (Phillips, 1959; see Crow and Ong, 1990, p. 225).

If it is a superconductor, then…





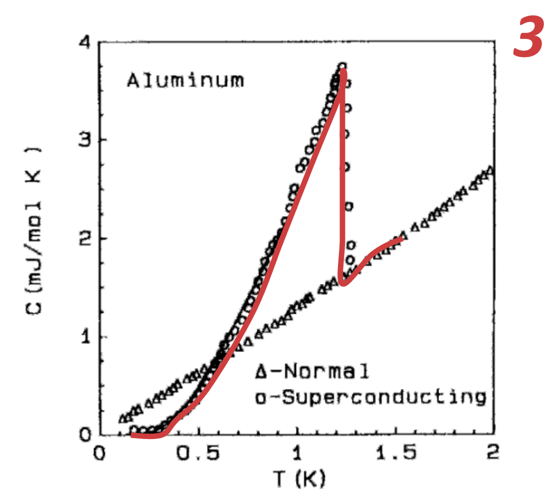


Figure 4.5 Specific heat jump in superconducting Al compared with the normal-state specific heat (Phillips, 1959; see Crow and Ong, 1990, p. 225).

Phenomenological theories: The London Theory

Drude model

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{h}\right] - \frac{\vec{v}}{\tau}$$

London model

$$\frac{d\vec{v}_s}{dt} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}\vec{v}_s\times\vec{h}\right]$$

$$\vec{v}_s = \vec{v}_s(x,y,z,t) \Rightarrow \frac{d\vec{v}_s}{dt} = \frac{\partial\vec{v}_s}{\partial t} + (\vec{v}_s\cdot\vec{\nabla})\,\vec{v}_s = \frac{\partial\vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) - \vec{v}_s\times\vec{\nabla}\times\vec{v}_s$$

$$\frac{\partial\vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) - \vec{v}_s\times\vec{\nabla}\times\vec{v}_s = -\frac{e}{m}\vec{E} - \frac{e}{mc}\vec{v}_s\times\vec{h}$$

$$\frac{\partial\vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) + \frac{e}{m}\vec{E} = \vec{v}_s\times\left(\vec{\nabla}\times\vec{v}_s - \frac{e}{mc}\vec{h}\right)$$

Phenomenological theories: The London Theory

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) + \frac{e}{m}\vec{E} = \vec{v}_s \times \left(\vec{\nabla}\times\vec{v}_s - \frac{e}{mc}\vec{h}\right)$$ *and define* $$\vec{Q} = \vec{\nabla}\times\vec{v}_s - \frac{e}{mc}\vec{h}$$

Rewrite with Q + Rotor

$$\frac{\partial}{\partial t}\vec{\nabla}\times\vec{v}_s + \frac{e}{m}\vec{\nabla}\times\vec{E} = \vec{\nabla}\times\left(\vec{v}_s\times\vec{Q}\right)$$

With the help of Maxwell

$$\frac{\partial}{\partial t}\vec{\nabla}\times\vec{v}_s + \frac{e}{m}\left(-\frac{1}{c}\frac{\partial \vec{h}}{\partial t}\right) = \vec{\nabla}\times\left(\vec{v}_s\times\vec{Q}\right)$$

$$\frac{\partial}{\partial t}\vec{Q} = \vec{\nabla}\times\left(\vec{v}_s\times\vec{Q}\right)$$

If at t = 0 $\vec{h} = 0,\ \vec{v}_s = 0 \Rightarrow \underline{\vec{Q} = 0} \Rightarrow \frac{\partial}{\partial t}\vec{Q} = 0$

$$\vec{\nabla}\times\vec{v}_s - \frac{e}{mc}\vec{h} = 0$$

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) + \frac{e}{m}\vec{E} = 0$$

London equations

The supercurrent density is defined as $\vec{j}_s = -n_s e \vec{v}_s$

$$\vec{\nabla} \times \vec{v}_s - \frac{e}{mc}\vec{h} = 0$$

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{\nabla}\left(\frac{1}{2}v_s^2\right) + \frac{e}{m}\vec{E} = 0$$

see F. London, Superfluids (1961), pp. 57-60

$$\vec{\nabla} \times \frac{m}{n_s e^2}\vec{j}_s = -\frac{\vec{h}}{c}$$

$$\frac{\partial \vec{v}_s}{\partial t} + \frac{e}{m}\vec{E} = 0$$

$$\frac{\partial}{\partial t}\frac{m}{n_s e^2}\vec{j}_s = \vec{E}$$

$$\vec{\nabla} \times \left(\Lambda \vec{j}_s\right) = -\frac{\vec{h}}{c}$$

1st London equation

$$\frac{\partial}{\partial t}\left(\Lambda \vec{j}_s\right) = \vec{E}$$

2nd London equation

London equations

Combine $\vec{\nabla}\times\left(\Lambda\vec{j}_s\right) - = -\frac{\vec{h}}{c}$ *and* $\vec{\nabla}\times\vec{h} = \frac{4\pi}{c}\vec{j}$

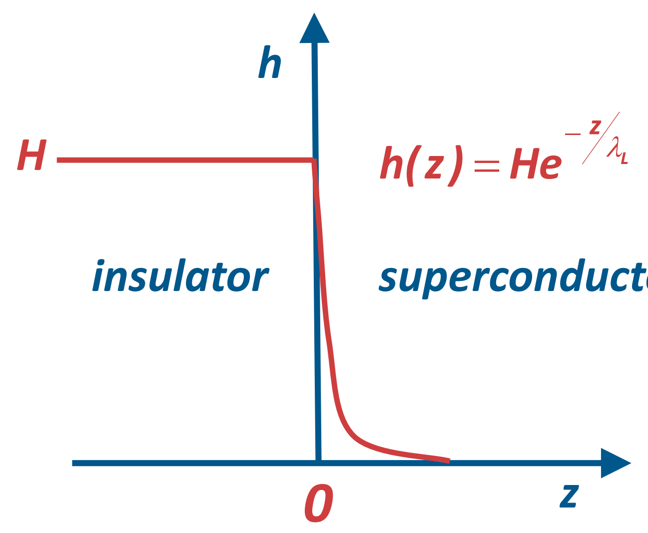
$$-\frac{\Lambda c^2}{4\pi}\vec{\nabla}\times\vec{\nabla}\times\vec{h} = \vec{h}$$

$$-\frac{\Lambda c^2}{4\pi}\left[\vec{\nabla}\left(\vec{\nabla}\cdot\vec{h}\right) - \nabla^2\vec{h}\right] = \vec{h}$$

$$\vec{h} - \lambda_L^2\nabla^2\vec{h} = 0$$

with $\lambda_L = \sqrt{\frac{\Lambda c^2}{4\pi}} = \sqrt{\frac{m^* c^2}{4\pi n_s e^{*2}}}$

penetration depth



Bibliography

Applications & Historical background

Rogalla & Kes
100 Years of Superconductivity

Phenomenology

Fosshein & Sudbø
Superconductivity: Physics and Applications
Chapter 1

London Equations

London
Superfluids – Macroscopic Theory of Superconductivity
Chapter B