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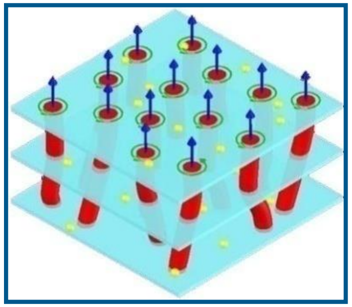
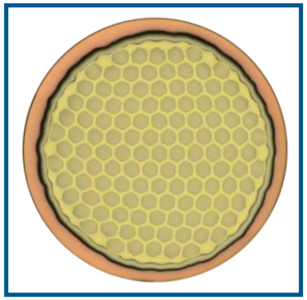
Superconductivity and its applications

Lecture 11





Carmine SENATORE

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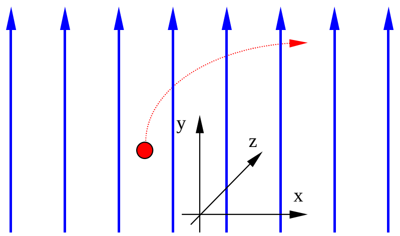
Previously, in lecture 10

Superconducting magnets, field shapes and winding configurations

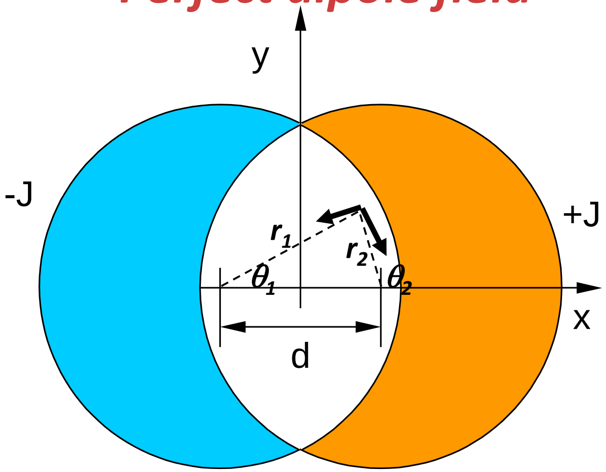
Transverse fields (particle accelerators)

Bending the beam

Uniform field (dipole)

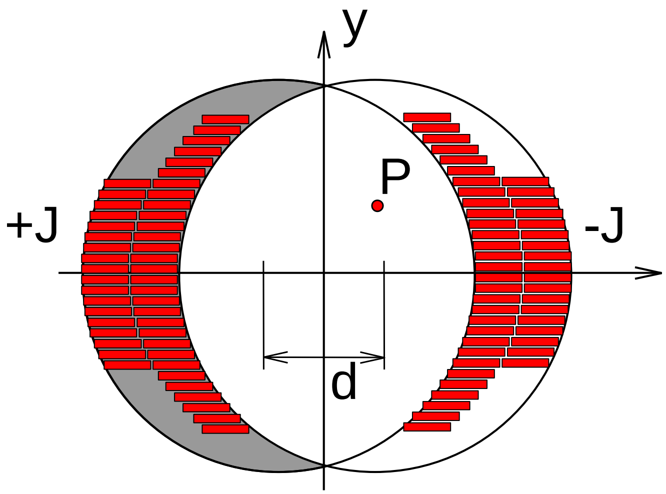


Perfect dipole field



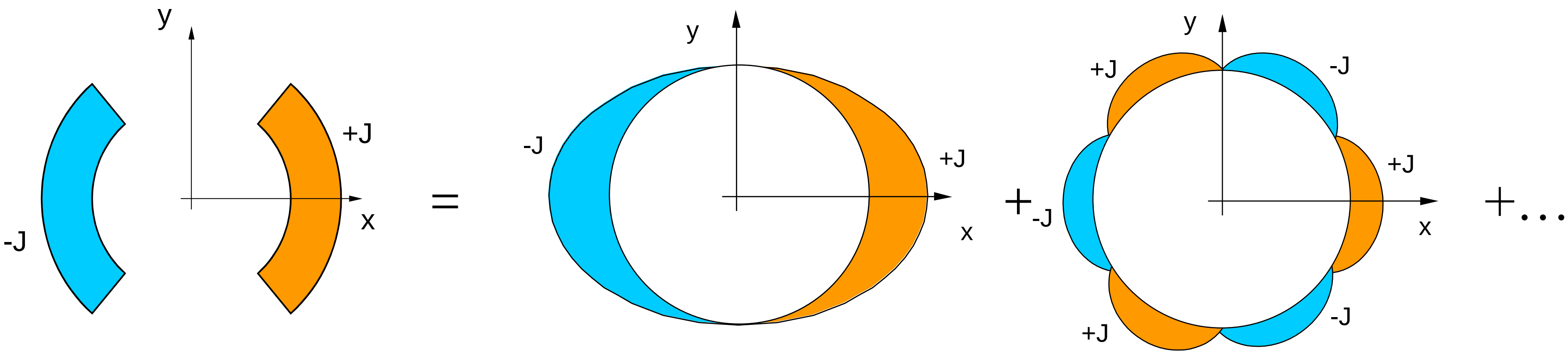
$$B_y = -\mu_0 \frac{Jd}{2} \qquad B_x = 0$$

Real dipole magnet



Previously, in lecture 10

Harmonic generation in real coils



Uniform current shell = $\cos\theta$ current + $\cos 3\theta$ current + …

Magnetic field = dipole field + sextupole term + …

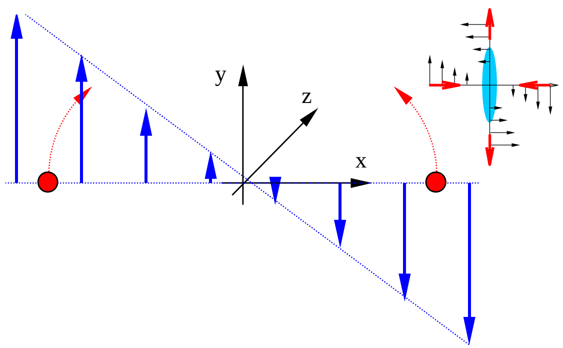
Previously, in lecture 10

Superconducting magnets, field shapes and winding configurations

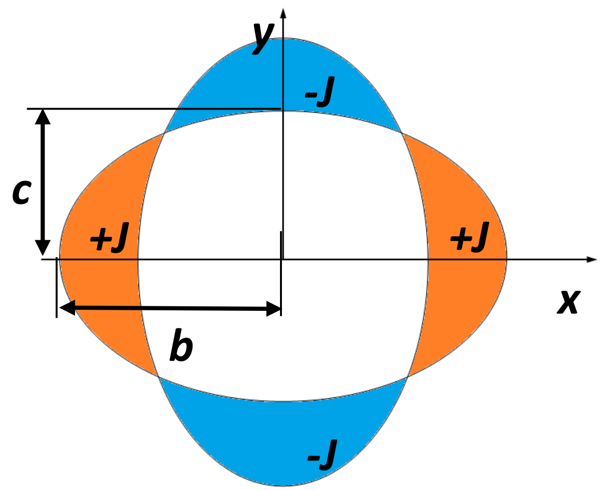
Transverse fields (particle accelerators)

Focusing the beam

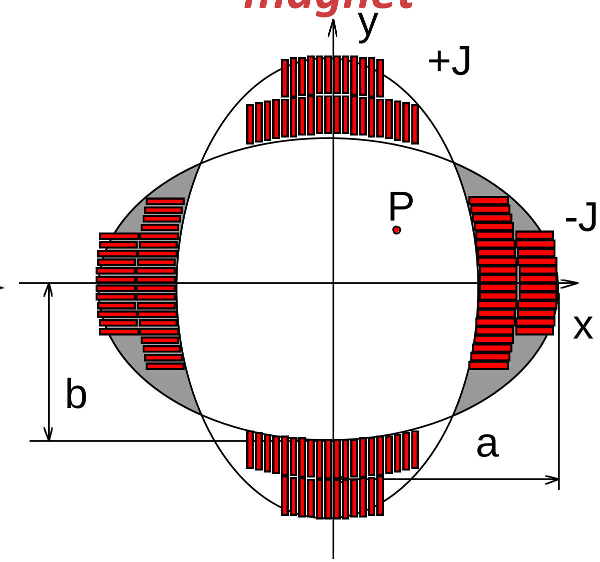
Gradient field (quadrupole)



Perfect quadrupole field



Real quadrupole magnet



$$B_y = \frac{\mu_0 J(b-c)}{b+c} y \qquad B_x = \frac{\mu_0 J(b-c)}{b+c} x$$

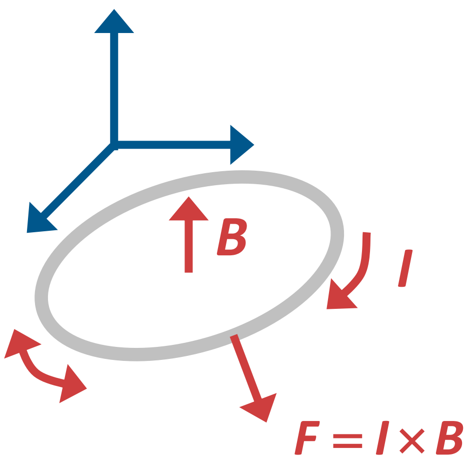
Uniform gradient $$g = \frac{\mu_0 J(b-c)}{b+c}$$

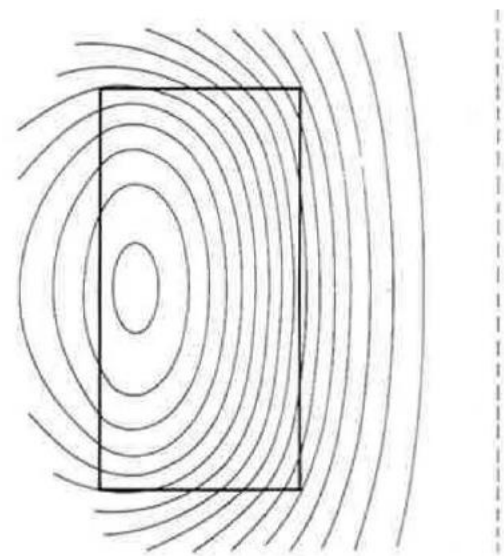
Previously, in lecture 10

Mechanical stresses on a SC wire in a magnet

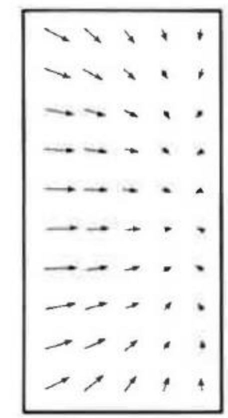
Magnetic forces within the magnet windings

$$\sigma_{hoop} = \frac{I}{S_{wire}} Br$$





magnetic lines of force



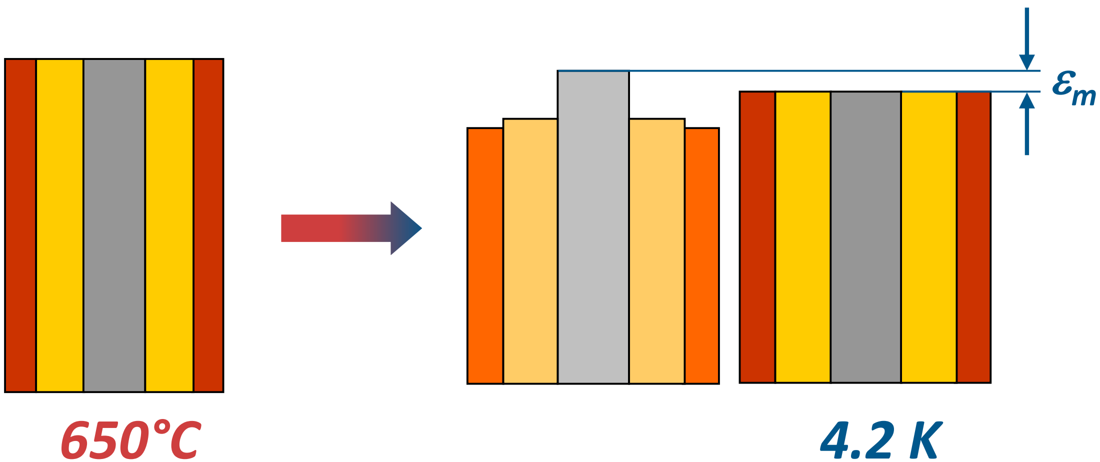
vectors of electromagnetic force per unit volume

In a winding adjacent turns will press on each other and develop a radial stress σ_r which modifies the hoop stress σ_θ

Previously, in lecture 10
Mechanical stresses on a SC wire in a magnet

All technical superconductors are composite

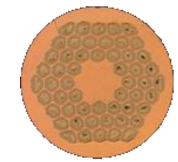
Mismatch in thermal contraction within the composite induces a precompression in the SC



When the SC is cooled at the operating temperature, its crystal structure deforms and this induces changes on T_c , B_{c2} (and thus on J_c)

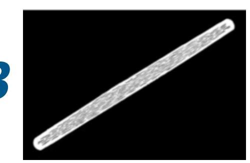
NbTi

Nb_3Sn

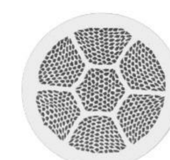


MgB_2

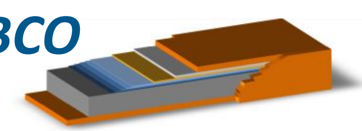
Bi2223



Bi2212

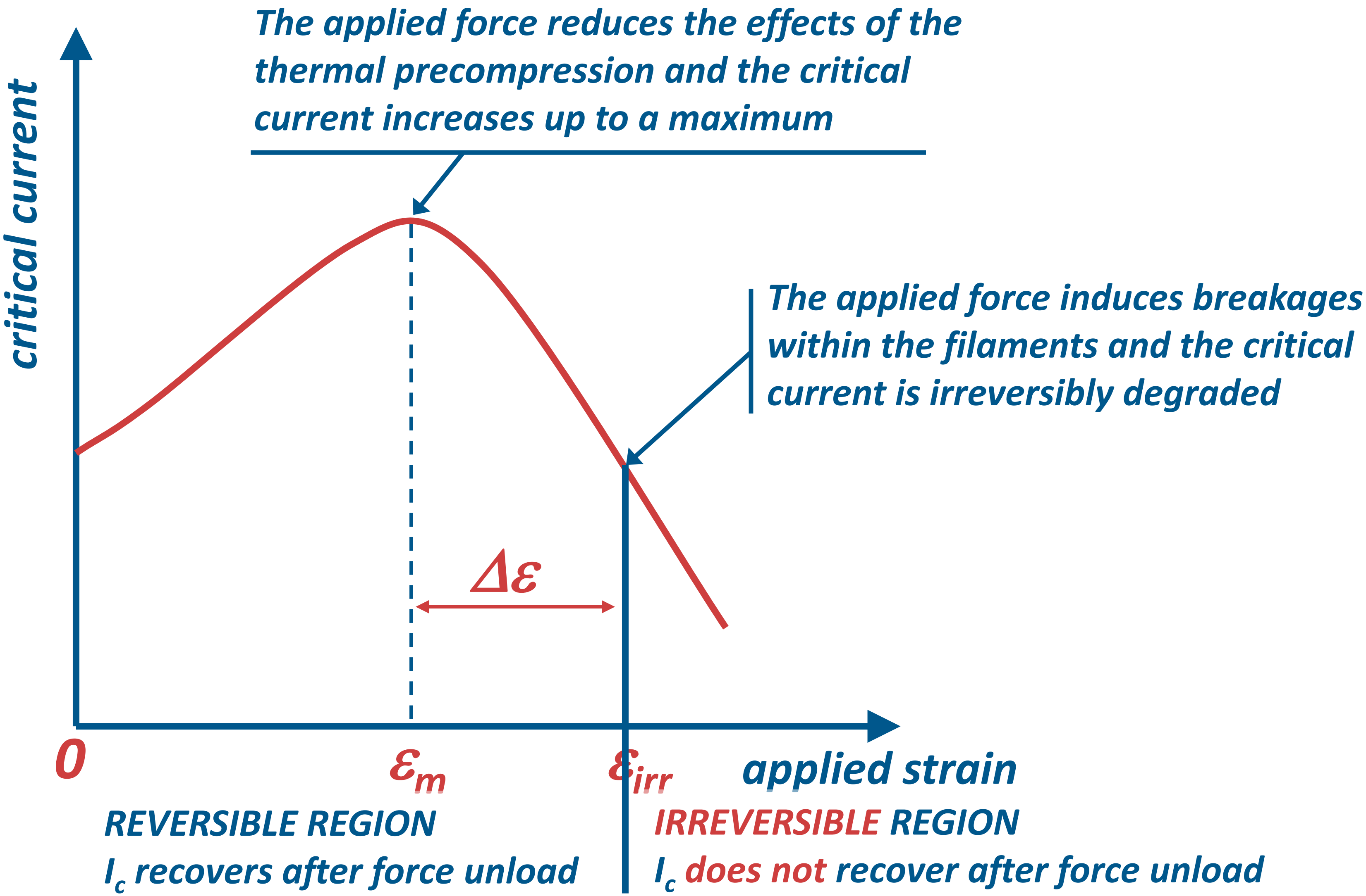


YBCO



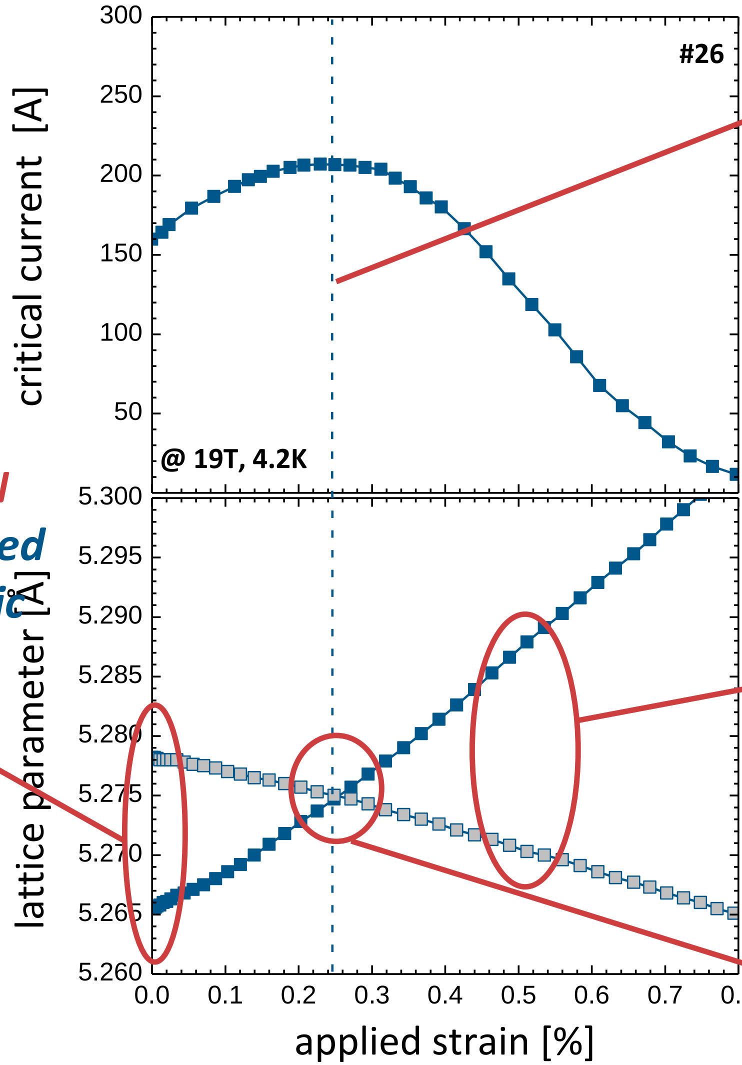
Previously, in lecture 10
Strain-induced changes in the critical current

Effects of the longitudinal strain



Previously, in lecture 10

Strain effects in Nb_3Sn wires: lattice parameters and I_c



MAXIMUM OF I_c
The maximum of the critical current occurs when the cubic cell is restored

ZERO APPLIED STRAIN
Nb_3Sn is precompressed
Hydrostatic+Deviatoric

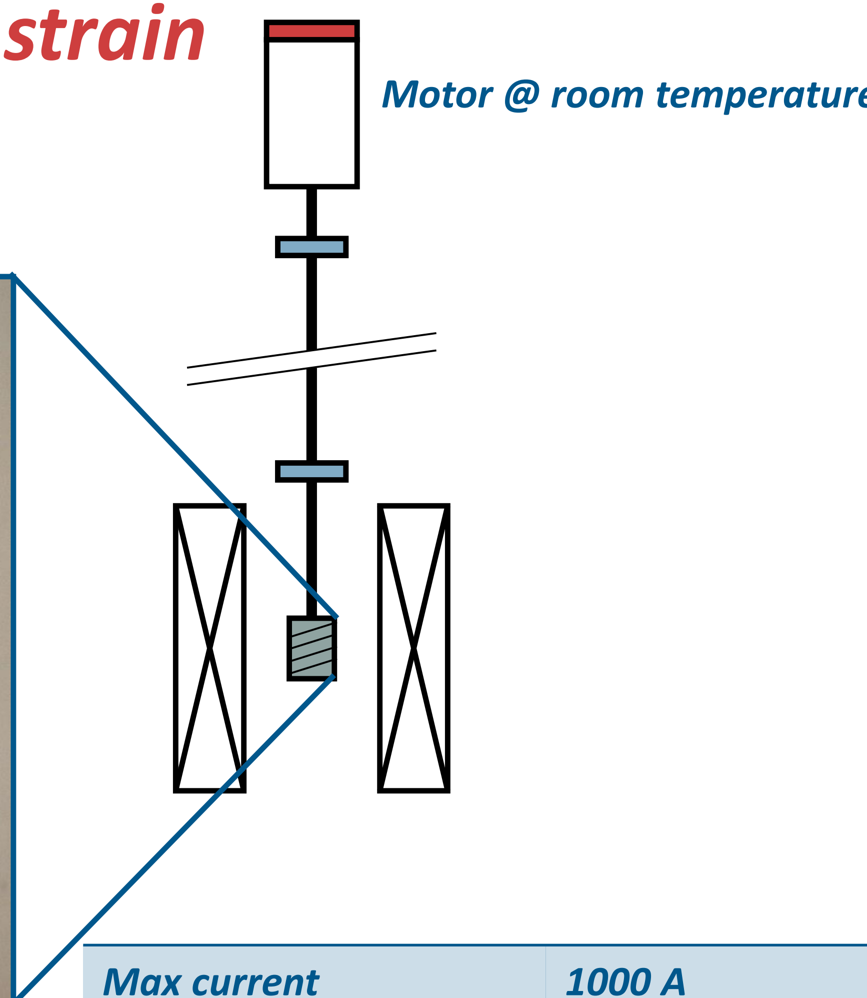
HIGH APPLIED STRAIN
Again Hydrostatic+Deviatoric

CROSSING POINT
Cubic cell recovered
Hydrostatic component still present

How to measure I_c vs. axial strain

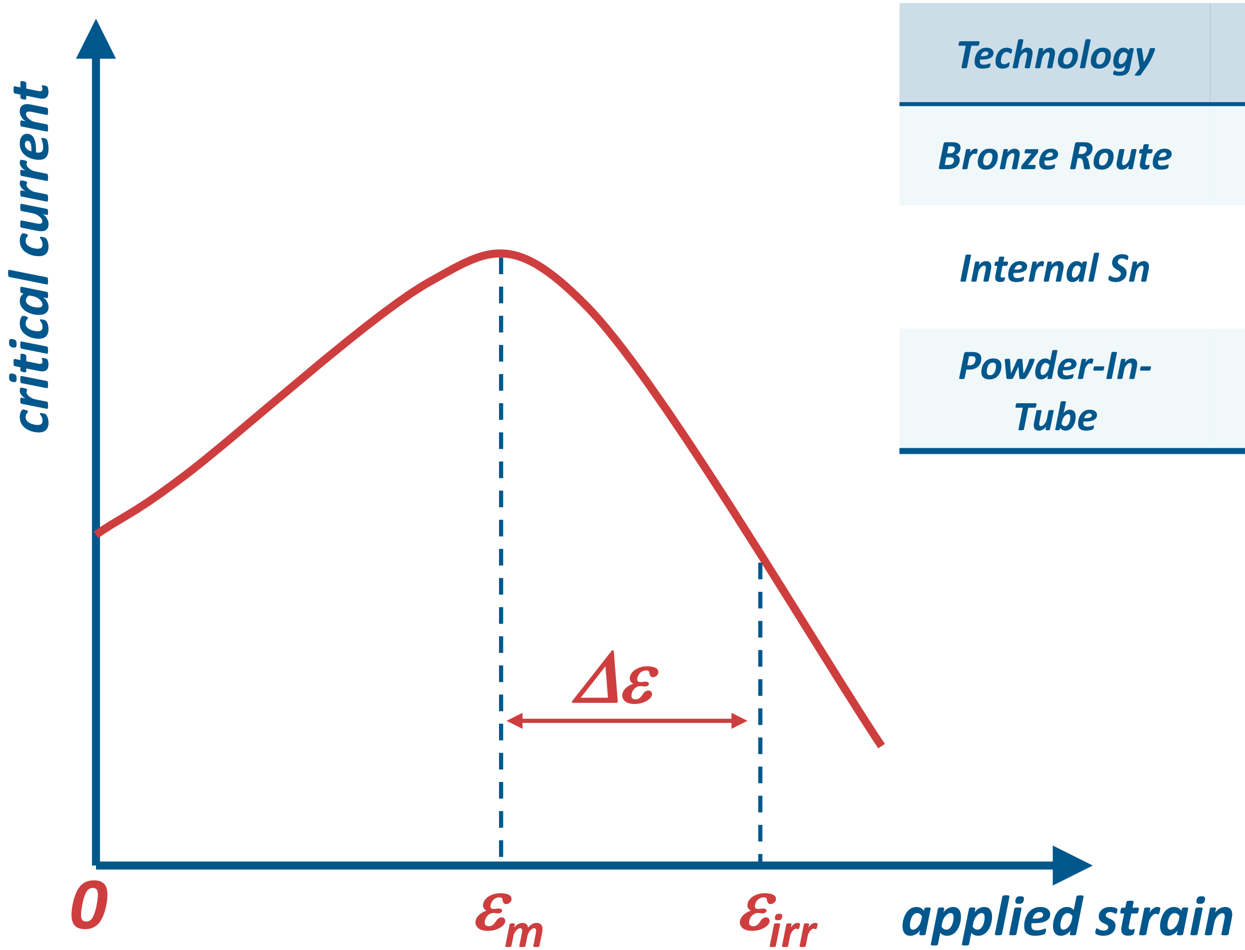
The WASP (Walters Spring) probe

B. Seeber et al., Rev. Sci. Instr. 76 (2005) 093901



Max current	1000 A
Sample length	1.1 meter
Voltage taps distance	126 mm
Electrical field criterion	0.01 µV/cm
Strain (ε)	up to ± 1.2 %

Irreversible strain limit in Nb_3Sn wires

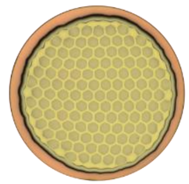


Technology	$\Delta\varepsilon = \varepsilon_m - \varepsilon_{irr}$ [%]
Bronze Route	0.4 – 0.6
Internal Sn	0.05 – 0.2
Powder-In-Tube	0.15 – 0.3

Industrial fabrication of Nb_3Sn wires

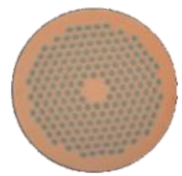
Three technologies have been developed at industrial scale

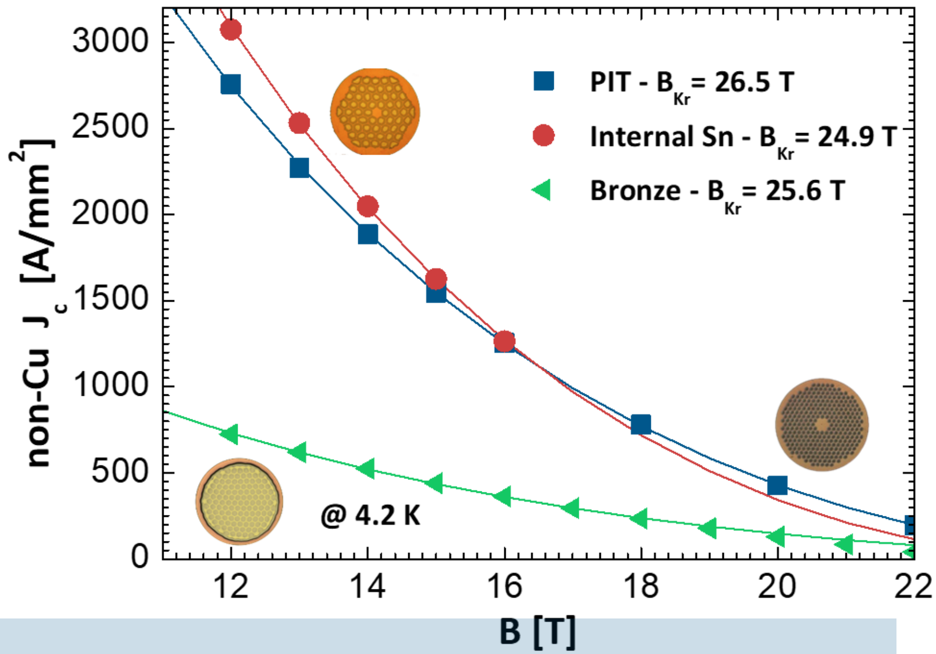
- ***Bronze route***
  Sn source = CuSn bronze



- ***Internal Sn diffusion***
  Sn source = Sn rod

- ***Powder in tube***
  Sn source = $NbSn_2$ powders

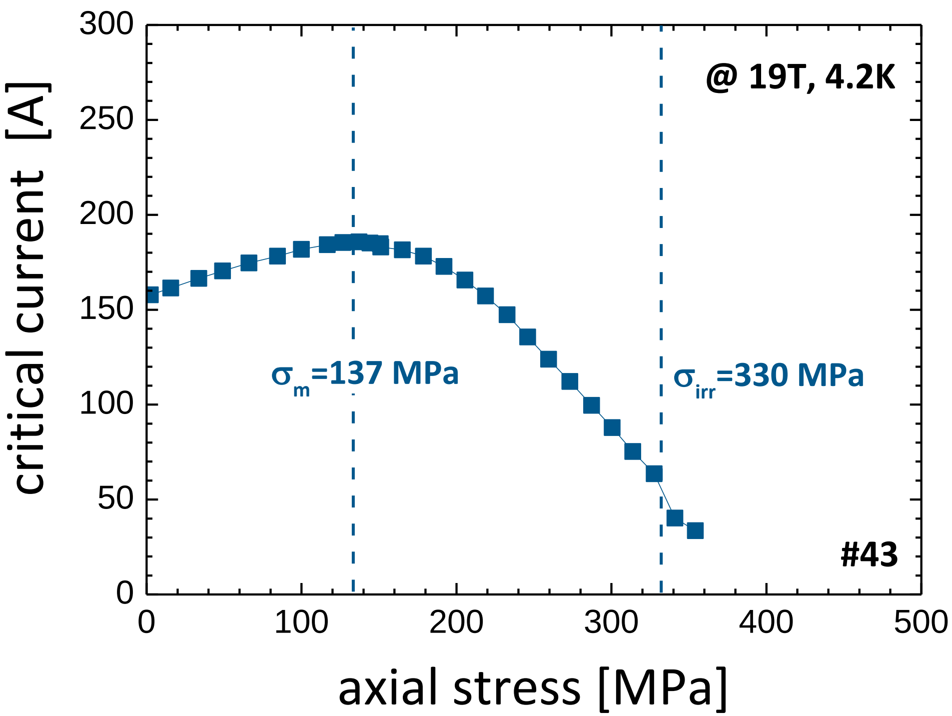


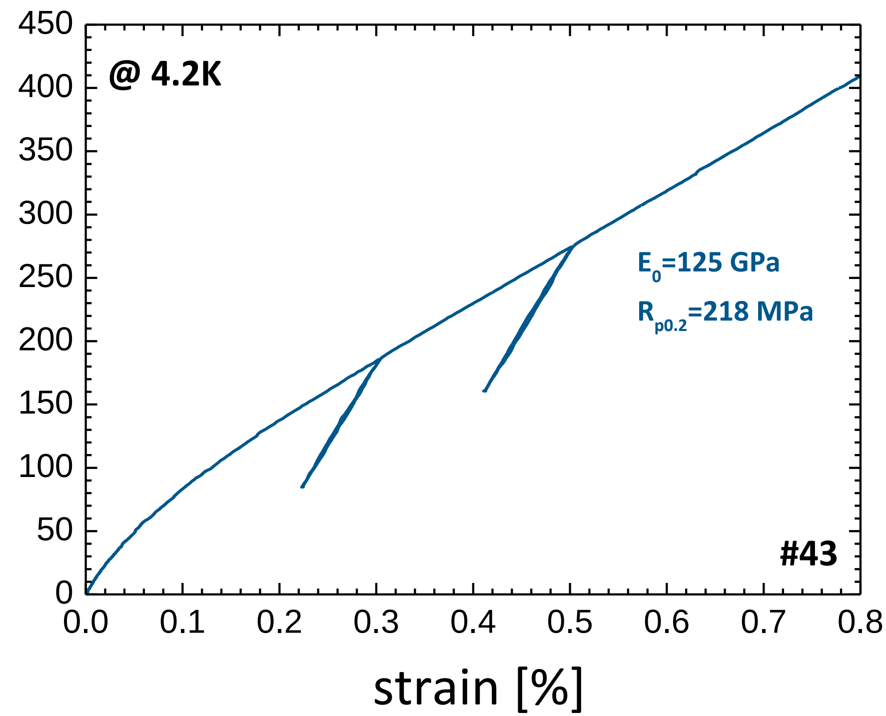


Technology	*Filament size*	*non-Cu J_c(12T,4.2K)*	
Bronze Route	1-5 µm	750 A/mm²	*Fine filaments ⇒ low hysteretic losses* *Bronze matrix ⇒ high mechanical strength* *Low Sn content ⇒ columnar grains, low J_c*
Internal Sn	20-100 µm	3’200 A/mm²	*High Sn content + Fine grains ⇒ very high J_c* *Large filaments ⇒ high hysteretic losses*
Powder-In-Tube	20-100 µm	2’700 A/mm²	*Fine grain region ⇒ high J_c* *Large grain region ⇒ very low J_c* *Large filaments ⇒ high hysteretic losses*

From I_c vs. strain to I_c vs. stress

Bronze Route wire

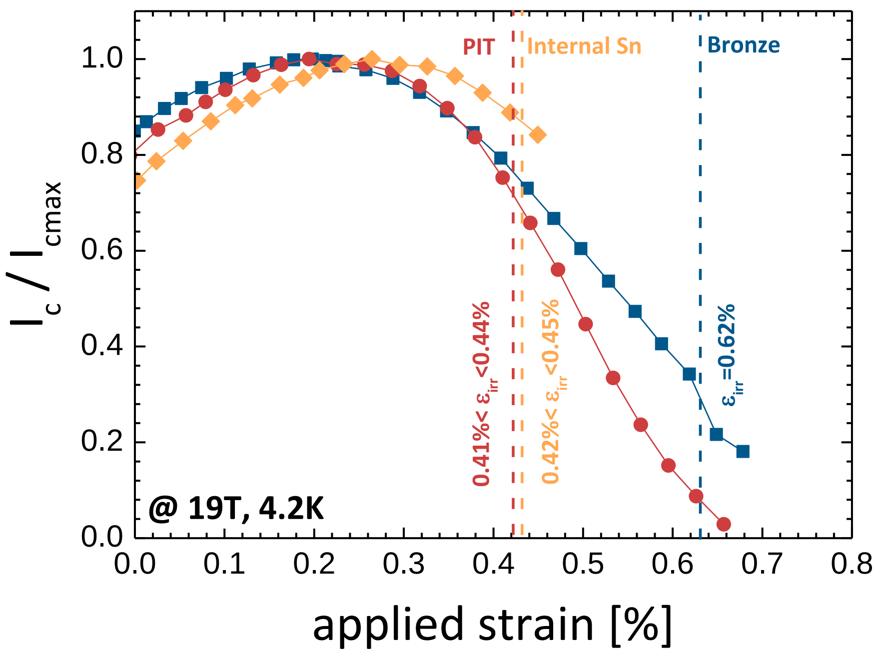




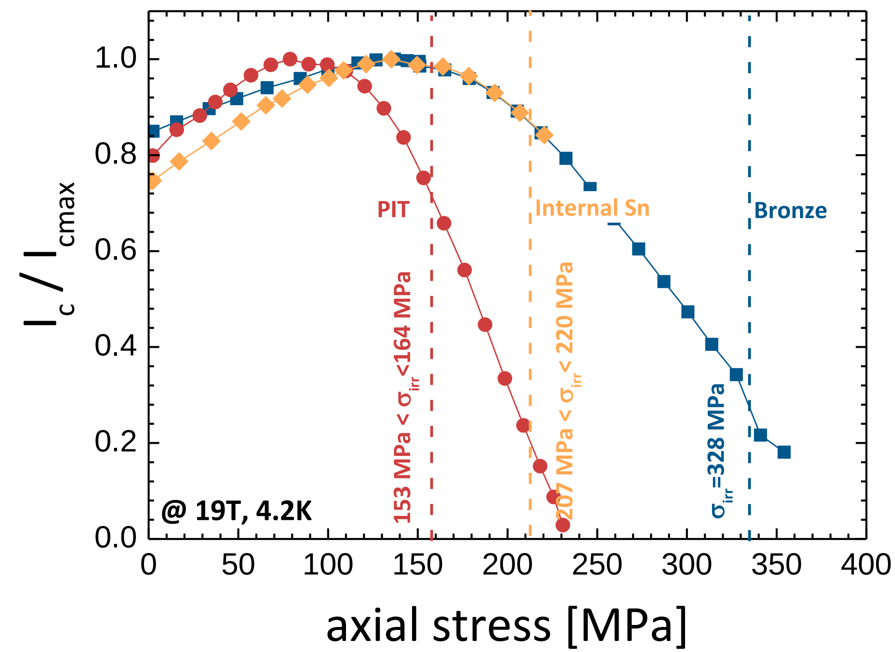
Bronze Route, Internal Sn and PIT

Reversible behaviour and irreversible limit

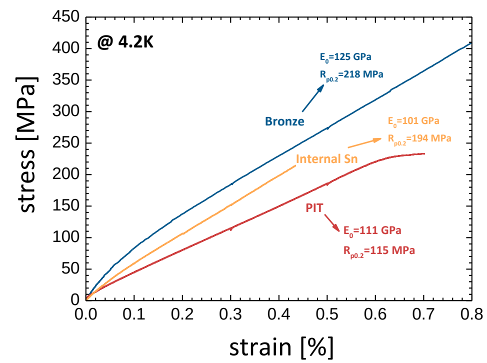
I_c vs. axial strain



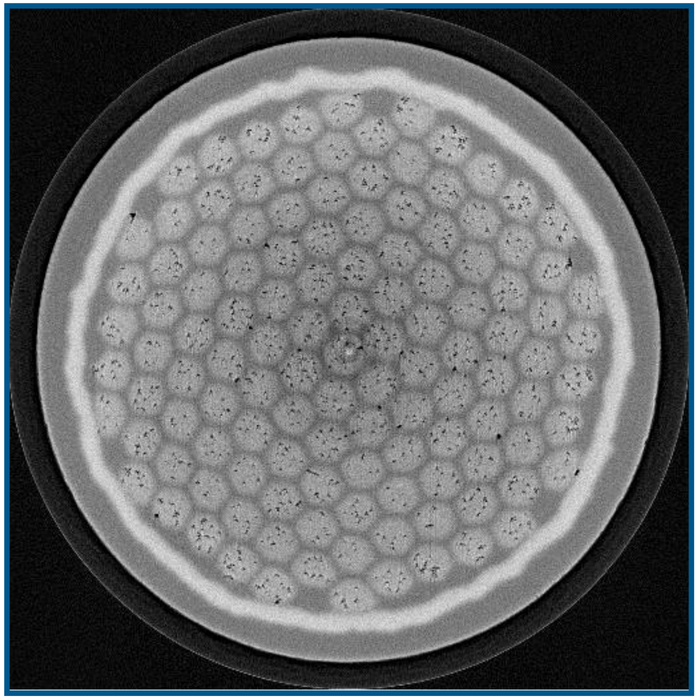
I_c vs. axial stress



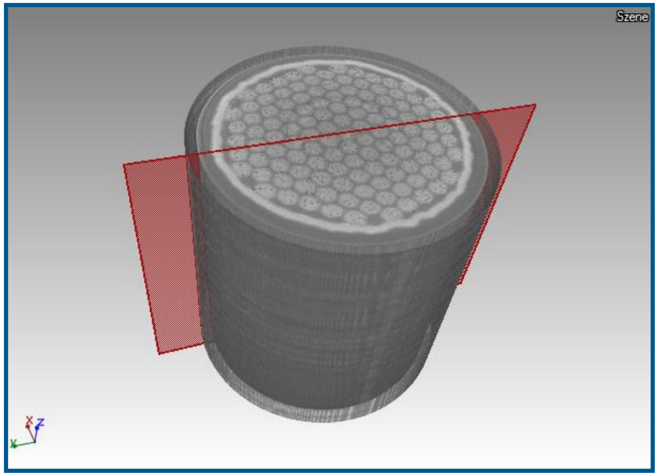
Technology	σ_{irr}
Bronze Route	330 MPa
Internal Sn	210 MPa
Powder-In-Tube	150 MPa



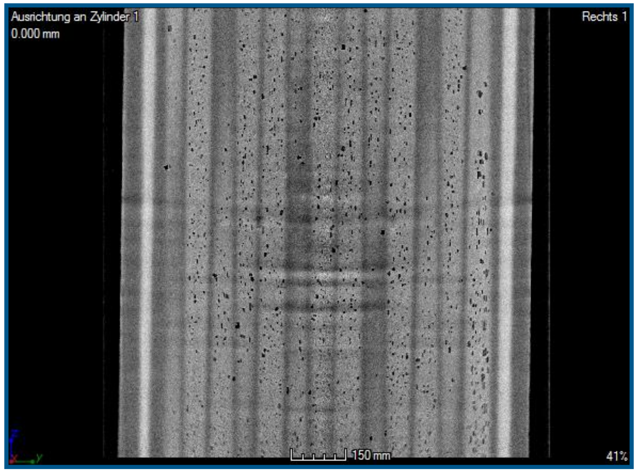
Microtomography and Analysis of Voids



Bronze wire

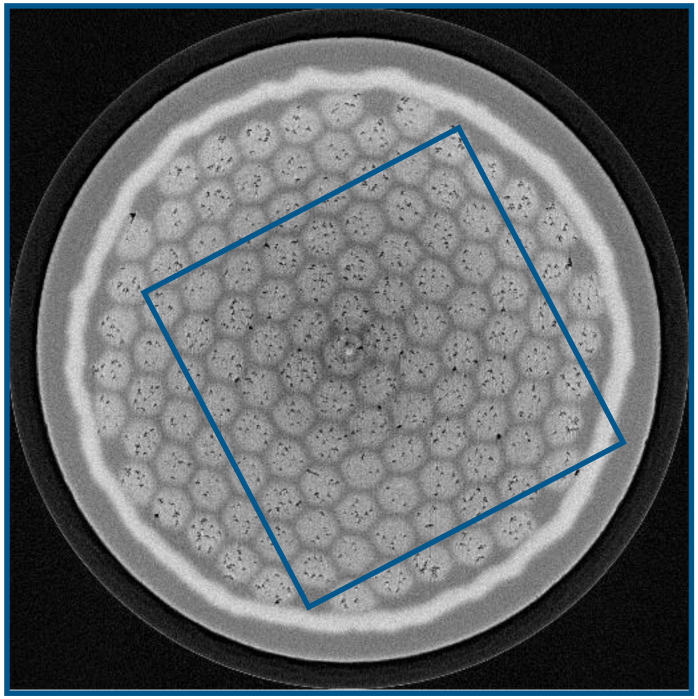


Virtual longitudinal cut

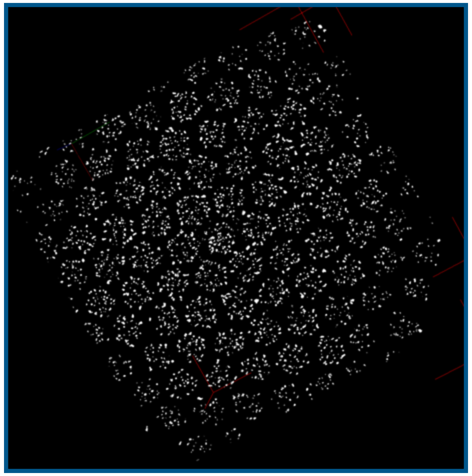


Experiments performed @ ESRF - Grenoble

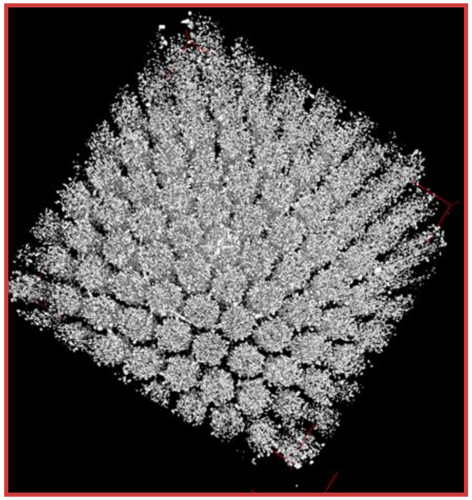
Microtomography: Visualize the Voids



Bronze wire

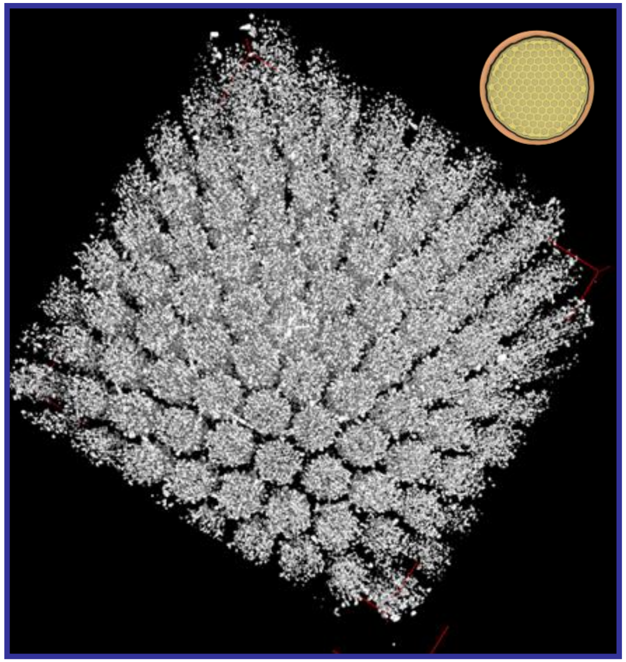


2D view of voids



3D view of voids

Void morphology and irreversible limit

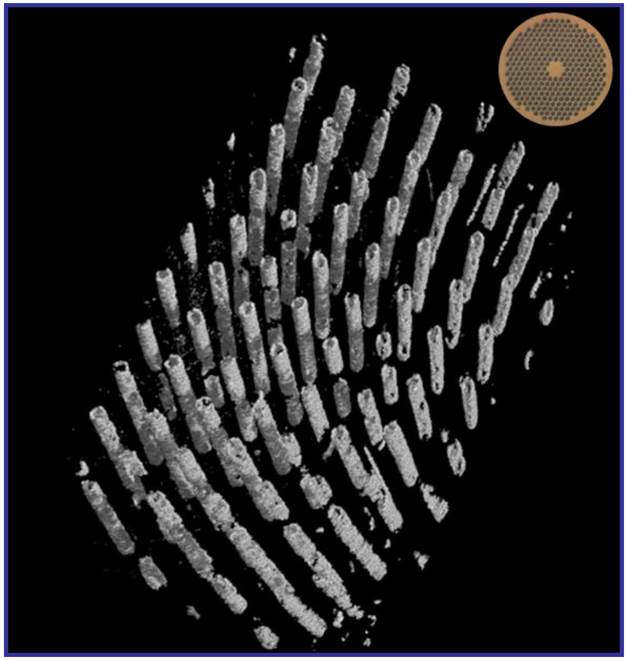


Bronze wire

σ_{irr} = 330 MPa

Internal Sn

σ_{irr} = 210 MPa



PIT

σ_{irr} = 120 MPa

From the wire to the magnet

Superconducting wire
- Superconductor
- Matrix metal
- Non-matrix metal

Superconducting magnet
- Superconducting wire
- Reinforcing elements and structure
- Insulation and filling materials
- (Coolant)

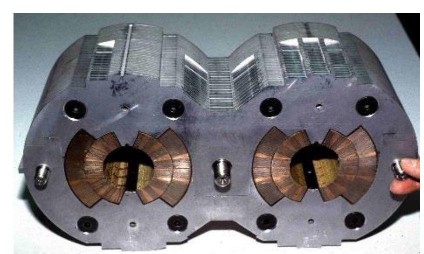
Cooling methods for superconducting magnets

The winding pack is dense, with no cryogen penetration, resulting in overall current densities in the winding significantly greater than those of bath-cooled cryostable magnets

Wet Magnets		
Cooling Method	*Cooling-Conductor Coupling*	*Heat Transfer*
Bath-cooled, "cryostable"	Good; entire conductor	Convective
Bath-cooled, "adiabatic"	Essentially nonexistent	Conductive
Force-cooled, "cryostable"	Good; entire conductor	Convective
Force-cooled, "quasi-stable"	Close proximity, but indirect	Conductive
Dry Magnets		
Cryocooled, "quasi-stable"	Indirect	Conductive

Cable-In-conduit, as for the ITER magnet

Dense winding, as for the accelerator magnets



Stability criteria for superconducting magnets

The power density equation governing the temperature of a superconductor unit volume is

$$\underbrace{C\frac{\partial T}{\partial t}}_{\text{Thermal Inertia}} = \underbrace{\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right)}_{\text{Conduction}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \underbrace{p_{initial}}_{\text{Thermal disturbance}} + \cancel{\underbrace{p_{cooling}}_{\text{Direct heat transfer to cryogen}}}$$

In an adiabatic magnet the last term can be neglected and the stability condition is defined by

$$\underbrace{\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right)}_{\text{Conduction}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \underbrace{p_{initial}}_{\text{Thermal disturbance}} = 0$$

Stability criteria for superconducting magnets

$$\underbrace{C\frac{\partial T}{\partial t}}_{\text{Thermal Inertia}} = \underbrace{\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right)}_{\text{Conduction}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \underbrace{p_{initial}}_{\text{Thermal disturbance}}$$

Table 6.2: Heat Capacities of Substances in Superconducting Magnets

Superconductor Operating Temperature Range	⟸ NbTi($T_c=9.8$ K) ⟹						
	⟸ MgB$_2$ ($T_c=39$ K) ⟹						
	⟸ YBCO ($T_c=93$ K) ⟹						
	$C_p(T)$ [J/cm^3 K]						
Material	2 K	4 K	10 K	20 K	30 K	50 K	90 K
Copper	0.00025	0.00089	0.0076	0.067	0.236	0.857	2.07
NbTi	0.00018	0.0014	0.022	—	—	—	—
MgB$_2$	0.000040	0.00032	0.00181	0.0081	0.0242	—	—
YBCO	0.000086	0.0007			0.120	0.454	1.12
Stainless steel	0.0014	0.003	0.01	0.04	0.1	0.4	1.5
Epoxy	0.00008	0.00066	0.014	0.080			

Because of the low specific heat at low temperatures, a high thermal conductivity of the superconducting composite is necessary to guarantee the stability

Table 6.3: Thermal Conductivities of Substances in Superconducting Magnets

Superconductor Operating Temperature Range	⟸ NbTi($T_c=9.8$ K) ⟹						
	⟸ MgB$_2$ ($T_c=39$ K) ⟹						
	⟸ YBCO ($T_c=93$ K) ⟹						
	$k(T)$ [W/cm K]						
Material	2 K	4 K	10 K	20 K	30 K	50 K	90 K
Copper	2	4.2	8.5	15	15	9	5
NbTi	0.0006	0.0017	0.0057	—	—	—	—
MgB$_2$			0.024	0.068	0.110	—	—
YBCO	0.020	0.080	0.120	0.225	0.250	0.240	0.125
Stainless steel	0.001	0.0027	0.009	0.02	0.035	0.057	0.088
Epoxy	0.0001	0.0003	0.0012	0.0027	0.004	0.006	0.007

Thermal conductivity of copper is orders of magnitude higher compared to superconducting materials

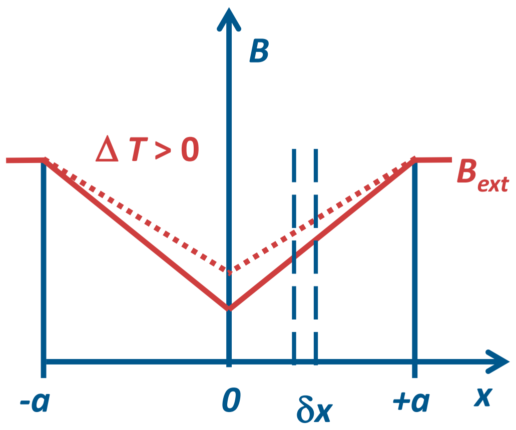
For this reason copper is always present in the layout of a superconducting wire

Thermal instabilities and Flux jumps (déjà-vu)

Flux motion after a thermal disturbance can further reduce the already low specific heat at low temperatures

If ΔQ_{ext} is the initial perturbation, the heat balance including the heat generated by the flux flow is

$$\Delta Q_{ext} + \frac{\mu_0 J_c a^2}{3\left(T_c - T_{op}\right)}\Delta T = c\Delta T$$



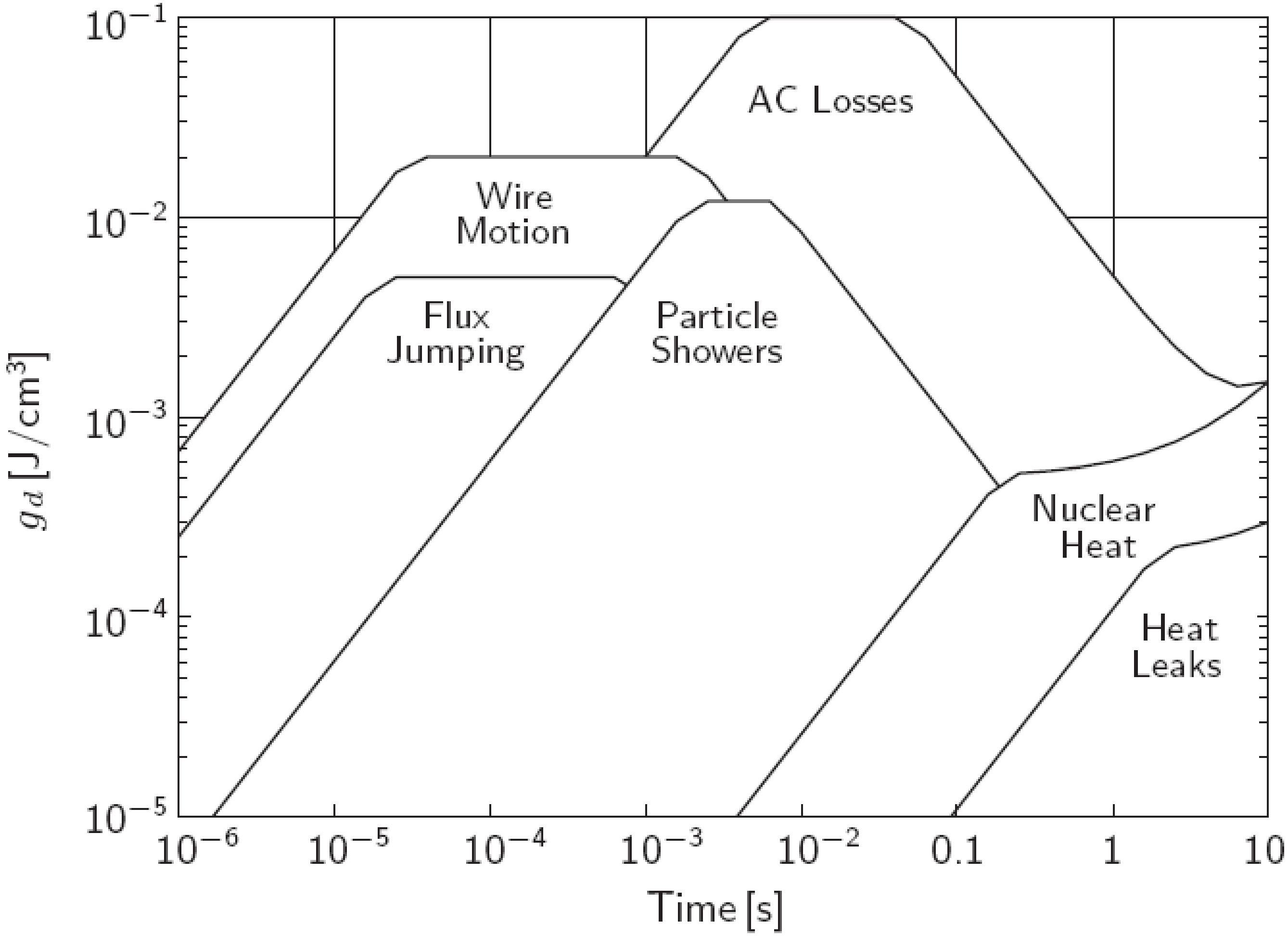
Because of the energy stored in the current, the effective specific heat is

$$c_{eff} = \frac{\Delta Q_{ext}}{\Delta T} = c - \frac{\mu_0 J_c^2 a^2}{3\left(T_c - T_{op}\right)}$$

c_{eff} can become zero $\Rightarrow$ The solution is to reduce the size of the filaments

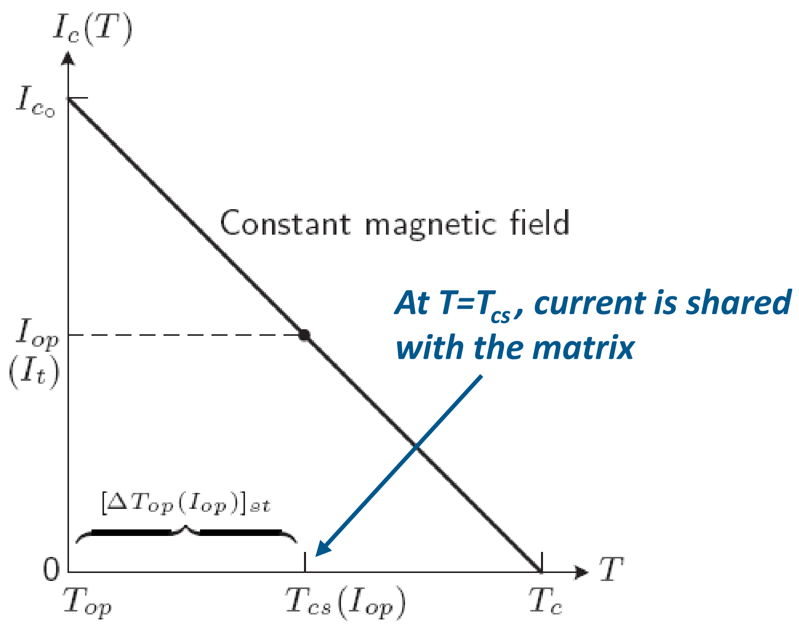
Disturbance spectra

$$\underbrace{C\frac{\partial T}{\partial t}}_{\text{Thermal Inertia}} = \underbrace{\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right)}_{\text{Conduction}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \underbrace{p_{initial}}_{\text{Thermal disturbance}}$$



Stability margin vs. Disturbance energy

The stability margin Δe is the maximum energy density that a composite superconductor carrying operating current I_{op} can absorb and still remain fully superconducting



$$\Delta e = \int_{T_{op}}^{T_{cs}(I_{op})} C(T)dT = \int_{T_{op}}^{T_{op}+\left[\Delta T_{op}(I_{op})\right]_{st}} C(T)dT$$

where

$$\left[\Delta T_{op}(I_{op})\right]_{st} = \left(T_c - T_{op}\right)\left(1 - \frac{I_{op}}{I_{c0}}\right)$$

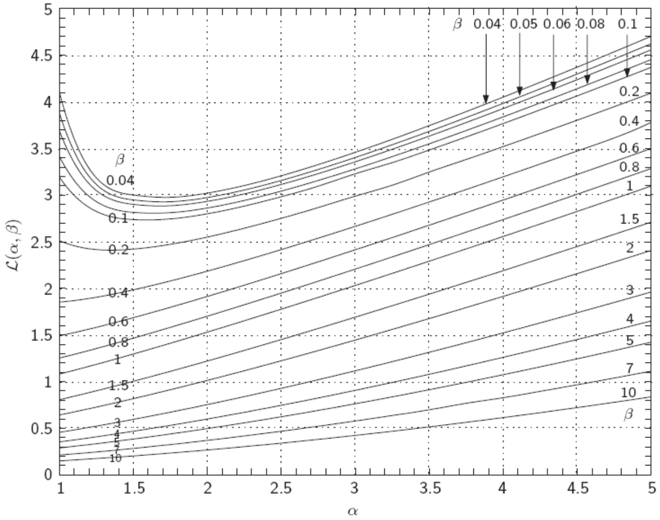
Table 6.4: Selected Values of T_{op}, ΔT_{op}, and Δe_h for LTS and HTS

LTS			*HTS*		
T_{op} [K]	$[\Delta T_{op}(I_{op})]_{st}$ [K]	Δe [J/cm^3]	T_{op} [K]	$[\Delta T_{op}(I_{op})]_{st}$ [K]	Δe [J/cm^3]
2.5	0.3	1.2×10^{-4}	4.2	25	1.6
4.2	0.5	0.6×10^{-3}	10	20	1.8
4.2	2	4.3×10^{-3}	30	10	3.7
10	1	9×10^{-3}	70	5	8.1

Hot spot and hot spot temperature

A magnet quench is initiated over a small winding volume, the so-called hot spot
The entire stored energy of the magnet may be dissipated over this hot spot, with the permanent damage of the magnet

The goal of any magnet protection is to limit the final temperature T_f below 300 K in case of quench



The stored magnetic energy E_m in a solenoid is

$$E_m = \frac{1}{2} L I^2$$

The inductance L of a solenoid (a_1,α,β) with N turns is

$$L = \mu_\circ a_1 \mathcal{L}(\alpha,\beta) N^2$$

The axial center field B_0 is given by

$$B_\circ = \frac{\mu_\circ N I}{2a_1(\alpha-1)\beta} F(\alpha,\beta) \quad \textit{with} \quad F(\alpha,\beta) = \beta \ln\left(\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right)$$

Hot spot and hot spot temperature (2)

For a solenoid (a,α,β), E_m may be related to its B_0 by

$$E_m = \frac{4a_1^3(\alpha-1)^2\beta^2\mathcal{L}(\alpha,\beta)}{F^2(\alpha,\beta)}\left(\frac{B_\circ^2}{2\mu_\circ}\right)$$

The total volume of the hot spot (resistive zone) V_r is given by

$$V_r = f_r V_w = f_r 2\pi a_1^3(\alpha^2-1)\beta$$

where V_w is the winding volume and f_r is the fraction of the hot-spot volume in the winding

If the magnet's total magnetic energy E_m is converted to heat adiabatically in the hot spot only, the average thermal energy density e_{mr} of the hot spot is given by

$$e_{mr} \equiv \frac{E_m}{V_r} = \frac{2(\alpha-1)\beta\mathcal{L}(\alpha,\beta)}{f_r\pi(\alpha+1)F^2(\alpha,\beta)}\left(\frac{B_\circ^2}{2\mu_\circ}\right)$$

What must be the minimum size of a hot spot volume in a solenoid to meet a requirement of $T_f \leq 300K$ is a key question of protection issue

Adiabatic heating under current discharge

How to estimate the hot spot temperature and design the winding to keep $T_f \leq 300K$

$$\underbrace{C\frac{\partial T}{\partial t}}_{\text{Thermal Inertia}} = \cancel{\underbrace{\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right)}_{\text{Conduction}}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \cancel{\underbrace{\rho_{initial}}_{\text{Thermal disturbance}}}$$

The heat balance per unit of volume is

$$J^2(t)\rho(T)dt = C(T)dT$$

Rearranging and integrating

$$\int_0^\infty J^2(t)dt = J_{op}^2 t_D = \int_0^{T_f}\frac{C(T)}{\rho(T)}dT = U\left(T_f\right)$$

t_D is a characteristic time for the current decay after a quench

To keep T_f low, t_D has to be short (J_{op} is given)

The shorter is t_D, the larger must be the hot spot volume

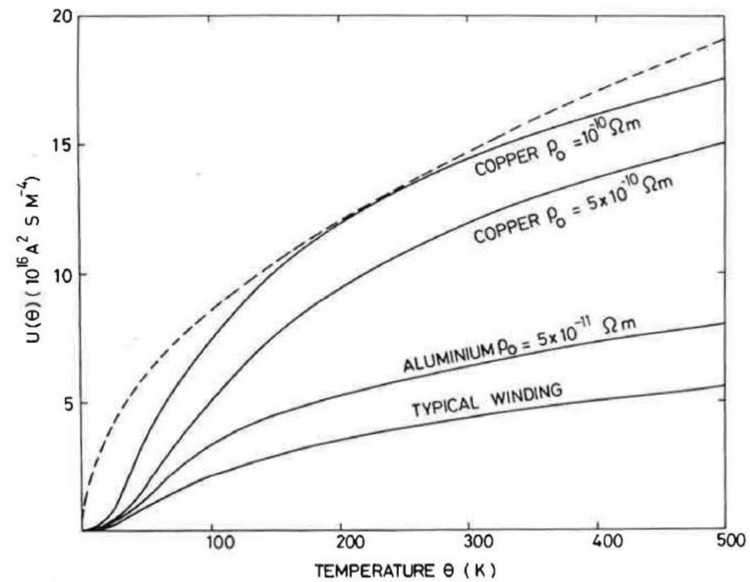


Fig. 9.1. The function $U(\theta)$ for various conductors and for a typical small magnet winding containing 47 per cent copper, 23 per cent NbTi, and 30 per cent resin. The dotted line shows the approximation $U(\theta) \sim \theta^{\frac{1}{2}}$.

Hot spot volume fraction and temperature

Table 8.1: Hot Spot Volume Fraction, f_r, vs. Hot-Temperature, T_f Solenoid ($\alpha = 1.5$, $\beta = 2.0$) of $B_\circ$ 1.5 T–30 T

$B_\circ$ [T]	e_m [J/cm^3]	$T_i = 4$ K, $T_f = 200$ K, f_r (%)	$T_i = 4$ K, $T_f = 300$ K, f_r (%)	$T_i = 80$ K, $T_f = 200$ K, f_r (%)	$T_i = 80$ K, $T_f = 300$ K, f_r (%)
1.5	1.02	0.27	0.14	0.31	0.15
3.0	4.06	1.1	0.57	1.2	0.62
6.0	16.2	4.3	2.3	5.0	2.5
12	65.0	17	9.1	20	9.9
20	180	48	25	55	27
25	282	74	40	87	43
30	406	107	57	125	62

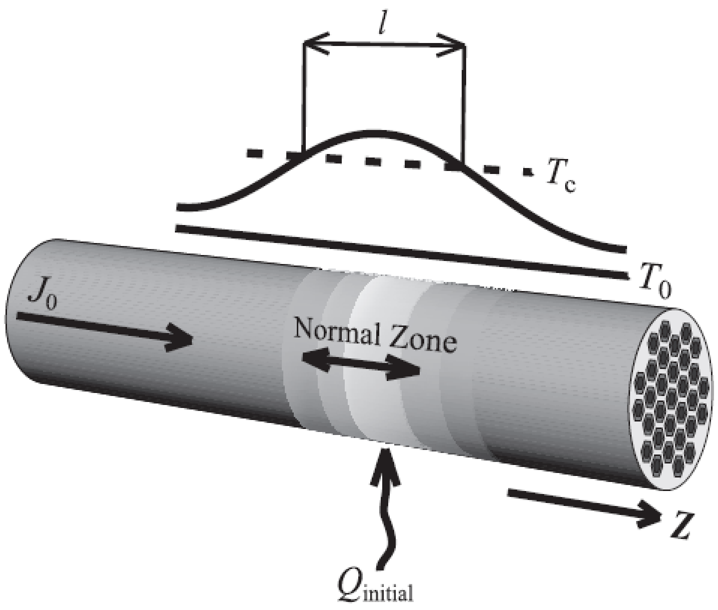
Hot spot volume fraction and temperature (2)

Table 8.2: "Permissible" Limits of T_f for Selected Materials

T [K]	*Remarks*
$\leq$200	Considered acceptable for LTS and HTS winding
320	Lowest melting point among indium solders
335	Cu-laminated YBCO: *no* I_c-degradation [8.12]
370	Cu-laminated YBCO: *slight* I_c-degradation [8.12]
380	Limits of Formvar insulation and Stycast 2850
400	Paraffin melts
430	Indium melts
493	50Sn-50Pb solder melts
720	Bi2223-Ag: *no* I_c-degradation [8.12]
800	Bi2223-Ag: I_c-degradation [8.12]

A quench in addition to damaging the winding thermally, may also do so mechanically by inducing localized strains.

Minimum Propagating Zone and Minimum Quench Energy



A thermal disturbance Q_{in} induces a temperature rise above T_c over a length l

The minimum length needed for this normal zone to propagate (for heat generation to exceed cooling) is defined as the minimum propagation zone l_{MPZ}

$$J^2 \rho A l_{MPZ} = 2\kappa A \frac{T_{cs} - T_{op}}{l_{MPZ}}$$

Minimum Propagating Zone $$l_{MPZ} = \sqrt{\frac{2\kappa\left(T_{cs} - T_{op}\right)}{J^2 \rho}}$$

Minimum Quench Energy $$MQE = A \int_{l_{MPZ}} dz \int_{T_{op}}^{T_{cs}} dT\; C\left(T\right)$$

$$\underbrace{C\frac{\partial T}{\partial t}}_{\text{Thermal Inertia}} = \underbrace{\frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right)}_{\text{Conduction}} + \underbrace{\rho J^2}_{\text{Joule Heating}} + \underbrace{p_{initial}}_{\text{Thermal disturbance}}$$

How to make a large MPZ and MQE

$$l_{MPZ} = \sqrt{\frac{2\kappa\left(T_{cs} - T_{op}\right)}{J^2 \rho}}$$

- *make thermal conductivity κ large*
- *make resistivity ρ small*

- *Superconductors have high ρ and low κ*
- *Copper have low ρ and high κ*
- *Mix copper and superconductor in a filamentary composite wire*
- *Superconductor in fine filaments for an intimate mixing*

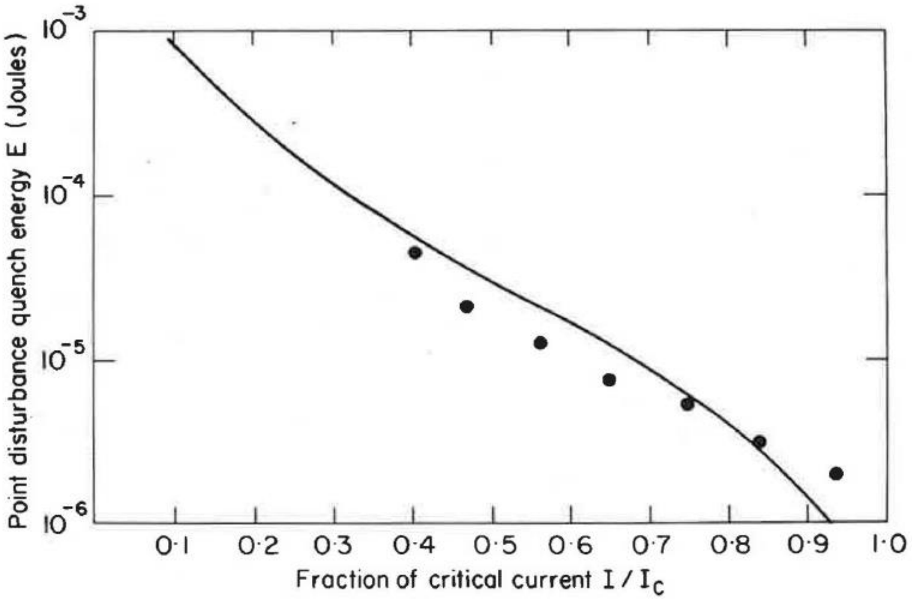
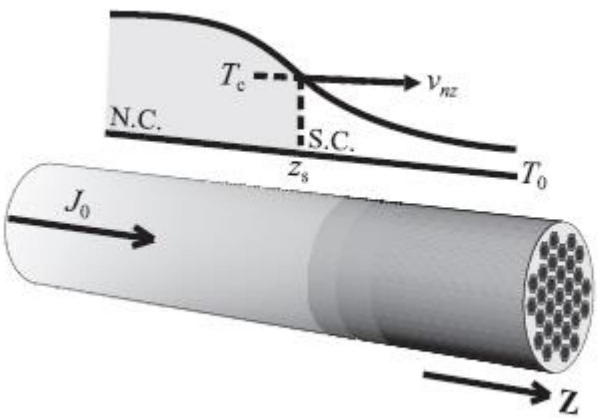


Fig. 5.13. Comparison between some experimental measurements of quench energy (points) and the predictions of Fig. 5.12 (solid line).

Normal Zone Propagation Velocity

Once a non-recovering normal zone is formed, it is desirable to make the normal-zone propagation (NZP) velocity "fast", to enlarge its f_r and limit the hot spot temperature



An expression for v_{NZ} can be derived by formulating two coupled diffusion equations for the normal- and superconducting zone

$$c_n \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\kappa_n \frac{\partial T}{\partial z}\right) + p_{\mathrm{diss}} \quad \text{(n.c.)};$$

$$c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\kappa_s \frac{\partial T}{\partial z}\right) \quad \text{(s.c.)},$$

By applying a coordinate transformation $z' = z - v_{NZ}t$, we obtain

$$v_{nz} c_n \frac{dT}{dz'} + \kappa_n \frac{d^2T}{dz'^2} + p_{\mathrm{diss}} = 0 \quad \text{(n.c.)};$$

$$v_{nz} c_s \frac{dT}{dz'} + \kappa_s \frac{d^2T}{dz'^2} = 0 \quad \text{(s.c.)}.$$

Normal Zone Propagation Velocity

By solving the coupled equations for v_{NZ}, we obtain

$$v_{NZ} = \sqrt{\frac{p_{diss}\kappa_n}{C_n C_s\left(T_{cs} - T_{op}\right)}}$$

that can be rewritten as

$$v_{NZ} = \frac{J}{C}\sqrt{\frac{\rho\kappa}{T_{cs} - T_{op}}}$$

Table 8.5: Selected Measured U_ℓ for LTS and HTS, Bare and Composite

Superconductor	Environment	T_{op} [K]	B_{ex} [T]	J [A/mm^2]	U_ℓ [mm/s]
Nb-Zr [8.26]	Liquid helium	4.2	0	100*	933
(single strand;				1000*	9330
no matrix metal)			6	100*	5345
		8.8	0	100*	1215
NbTi [8.36]	Liquid helium	4.2	0	420†	“Recovery”
(multifilamentary				840†	6800
composite)			4	420†	4660
				840†	18600
Nb_3Sn [8.39]	Adiabatic	4.2	0	630†	1830
(multifilamentary			6	315†	1490
composite)				630†	3720
Nb_3Sn [8.55]	Quasi-adiabatic	12	0	700†	510
(tape)		5.5	5	470†	525
Bi2223-Ag [8.55]	Quasi-adiabatic	40	0	230†	2
YBCO [8.64]	Adiabatic	46	0	10–15†	2–8
(coated) [8.69]	Adiabatic	77	0	3–15†	3–10
[8.73]	Adiabatic	77	0	65†	2.5
				115†	9
		40	0	115†	38
MgB_2 [8.70]	Quasi-adiabatic	4.2	4	26†	No NZP
(single strand;				78†	930
iron matrix)				212†	6000

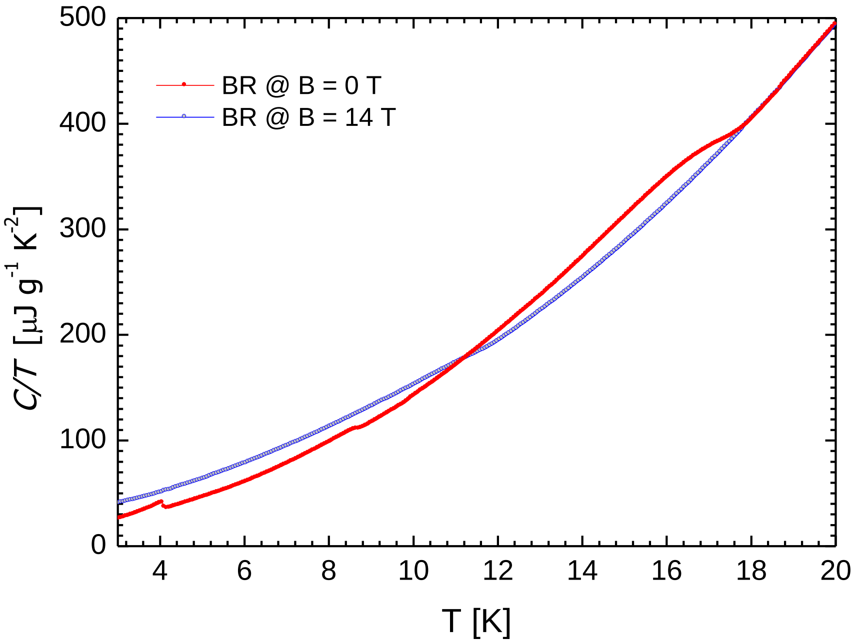
MPZ and NZPV

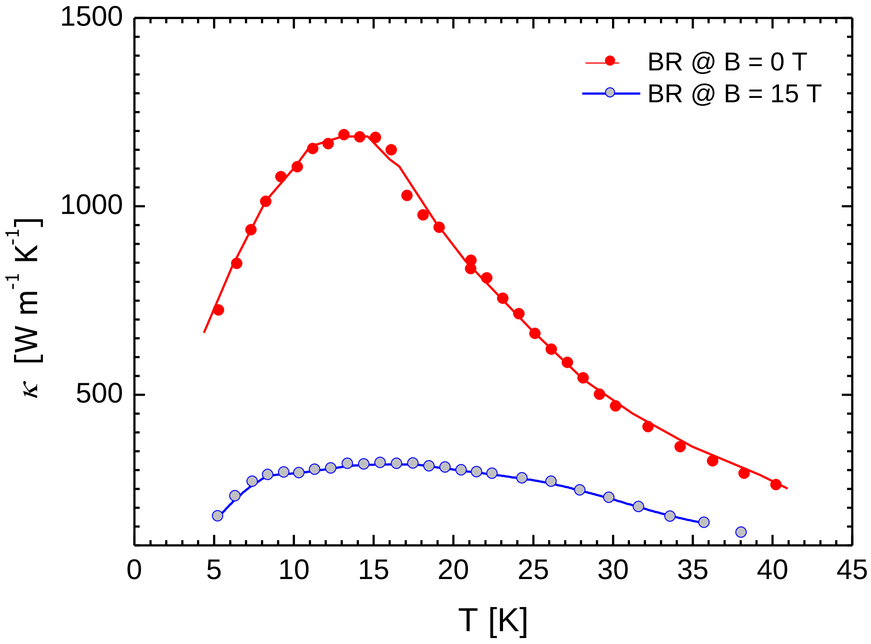
$$l_{MPZ} = \frac{1}{J}\sqrt{\frac{2\kappa}{\rho}\left(T_{cs} - T_{op}\right)}$$

$$v_{NZ} = \frac{J}{C}\sqrt{\frac{\rho\kappa}{T_{cs} - T_{op}}}$$

Specific Heat and Thermal Conductivity

Temperature and Field dependence for a Bronze Route Nb_3Sn wire



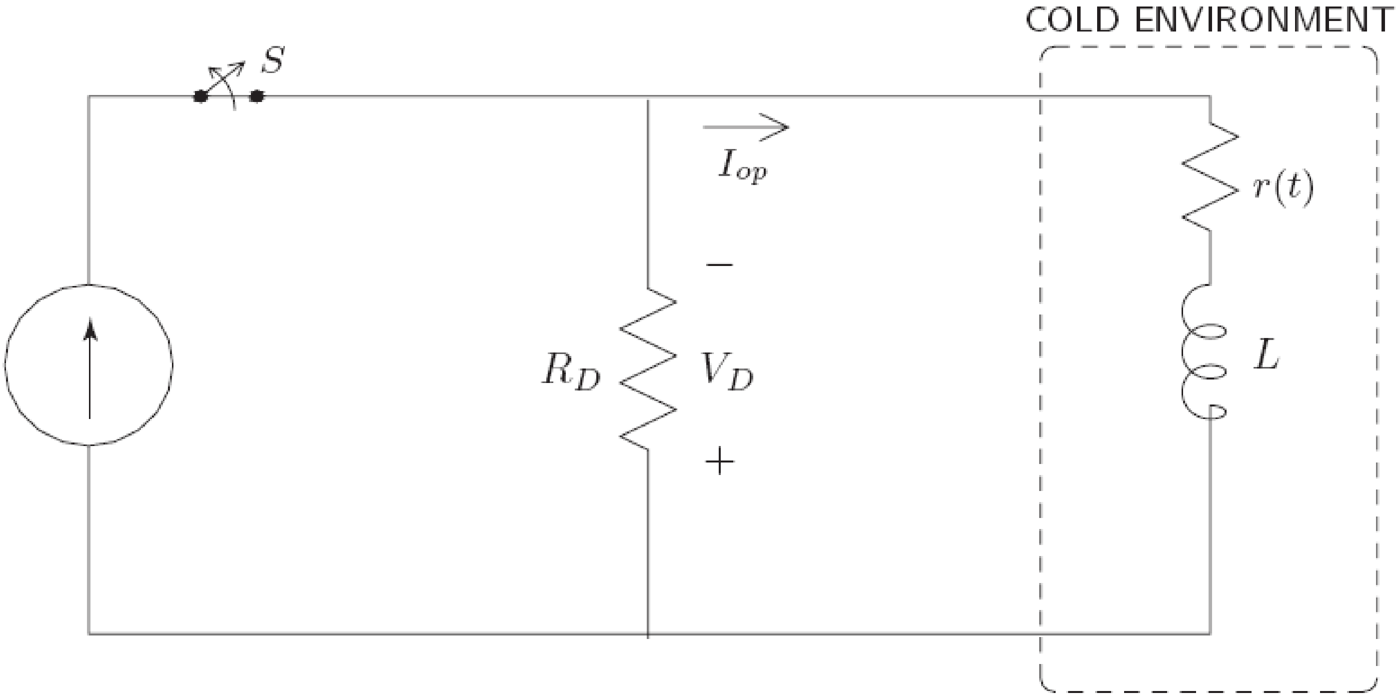


Specific heat is dominated by phonons and thus is weakly affected by the magnetic field

Thermal conductivity is dominated by electrons and thus is strongly affected by the magnetic field

How to protect a magnet in case of quench

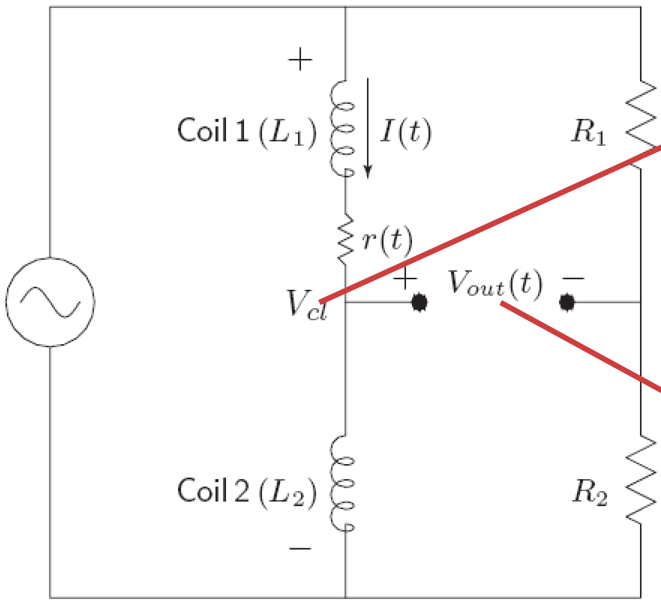
Active Protection Technique: Detect-and-Dump



A simple protection circuit with a switch S and external dump resistor R_D
When the start of a quench is detected, S opens and the current decays through R_D
If we make R_D larger than the internal quench resistance, it will dominate the current decay giving

$$I = I_{op} e^{-R_D t/L} = I_{op} e^{-t/t_D}$$

How to detect a quench - Basic Bridge Circuit



$$V_{cl}(t) = L_1 \frac{dI(t)}{dt} + rI(t) + L_2 \frac{dI(t)}{dt}$$

The current flowing in R_1 and R_2 is

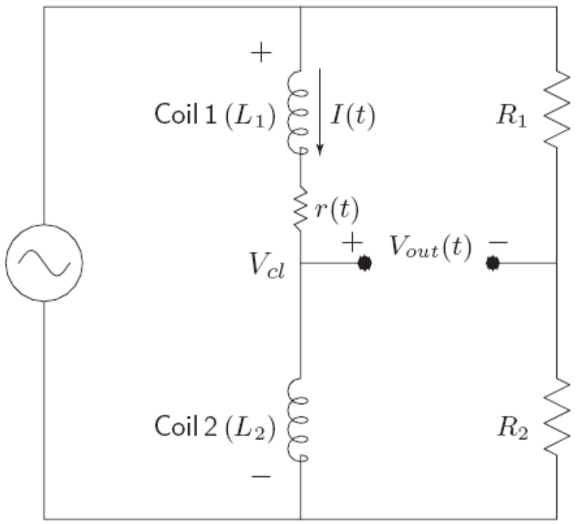
$$i_R(t) = \frac{V_{cl}(t)}{R_1 + R_2}$$

$$V_{out}(t) = L_1 \frac{dI(t)}{dt} + rI(t) - R_1 i_R(t)$$

V_{out} can be rewritten as

$$V_{out}(t) = L_1 \frac{dI(t)}{dt} + rI(t) - \frac{R_1}{R_1 + R_2}\left[L_1 \frac{dI(t)}{dt} + rI(t) + L_2 \frac{dI(t)}{dt}\right]$$

How to detect a quench - Basic Bridge Circuit



$$
\begin{aligned}
V_{out}(t) &= L_1 \frac{dI(t)}{dt} + rI(t) \\
&\quad - \frac{R_1}{R_1+R_2}\left[L_1 \frac{dI(t)}{dt} + rI(t) + L_2 \frac{dI(t)}{dt}\right] \\
&= \left(\frac{R_2}{R_1+R_2}\right) L_1 \frac{dI(t)}{dt} \\
&\quad - \left(\frac{R_1}{R_1+R_2}\right) L_2 \frac{dI(t)}{dt} + \left(\frac{R_2}{R_1+R_2}\right) rI(t)
\end{aligned}
$$

We design the circuit to make $V_{out}(t)$ proportional only to $rI(t)$

$$\left(\frac{R_2}{R_1+R_2}\right) L_1 \frac{dI(t)}{dt} - \left(\frac{R_1}{R_1+R_2}\right) L_2 \frac{dI(t)}{dt} = 0$$

This implies $R_2L_1 = R_1L_2$. In the case of quench we detect

$$V_{out}(t) = \left(\frac{R_2}{R_1+R_2}\right) rI(t)$$

Bibliography

Wilson
Superconducting Magnets
Chapter 5
Chapter 9

Iwasa
Case Studies in Superconducting Magnets
Chapter 4 Section 2
Chapter 6 Sections 1 & 2
Chapter 8 Sections 1, 2, 4 & 8